Soft Constrained Warm Start based MPC-PD Approach for Real Time Control of Underactuated Systems

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***Abstract*—Controller design for unstable underactuated systems has predominantly focused on fixed value control strategies. However, integrating the benefits of fixed value control with predictive control approaches remains a relatively under explored area for such systems. This article introduces a real time dual control strategy that combines Proportional Derivative (PD) control and Model Predictive Control (MPC) methods. The MPC uses a warm start and anticipates the future actuator movements, without constraint violation. The PD control provides an inner velocity control loop to reduce oscillations. By state augmentation, quadratic optimisation is implemented to find the optimum solution without violating the constraints. The proposed strategy has been implemented in real time on a rotary inverted pendulum system, designed to guide the arm along a trajectory while maintaining the pendulum upright. A comparative experimental study is conducted on this benchmark system, evaluating the proposed dual controller against a conventional MPC, with the proposed controller achieving better performance.**

Link to graphical and video abstracts, and to code: https://latamt.ieeer9.org/index.php/transactions/article/view/9660

***Index Terms*—Model predictive control, Nonlinear system, Warm start.**

I. Introduction

CONSTRAINT oriented motion for an unstable under actuated system restricts the system motion within particular trajectories. Due to actuator limitations, larger control input tends to cause actuator saturation, and as a result, the system output suffers from oscillations. The presence of these overshoots affects the system's positioning time, deteriorating the performance. The elimination of positioning errors is thus necessary. Several control strategies have been quoted in the literature, such as composite control based algorithm [1], observer based linear control [2], input-output feedback linearisation based control [3], Linear Quadratic Regulator (LQR) approach [4], [5], [6], [7], [8], [9], [10], pole placement technique [11], Proportional Integral Derivative (PID) control [12], [13], etc which extensively deals with handling the underactuated system dynamics.

Model Predictive Control (MPC) is one type of strategy that anticipates the future actuator movements without causing constraint violation [14]. The approach introduced in [15] for controlling biped locomotion uses a disturbance observer and a cascaded nonlinear MPC strategy. The linear MPC in [16] stabilises the cart pendulum system in the inverted position. [17] presents a controller that provides anti-windup conditioning to guarantee the optimality of the MPC problem. The work developed in [18] proposes a decomposition of the control problems into sets of independent MPCs of lower dimensions, which retains all information about the system. In nonlinear, open loop unstable systems, a cold start based MPC may fail to consistently produce convergent solutions for the optimisation problem. The warm start initiated in MPC, as discussed in [19], is a formulation that infuses previously computed LQR gains to improve the system stability and trajectory tracking. On the other hand, the learning method proposed in [20] uses previously computed data to generate spatial warm starts for the MPC optimisation problem.

Additionally, multiple techniques have been discussed in the following literature to reduce the oscillatory responses during system movement. Open loop strategies are popular as they only require a fairly accurate system model to ensure satisfactory performance. The offline minimum time trajectory planning approach introduced in [21] aids in jerk reduction. Damping of oscillations based on system inversion has been proposed in [22], while [23] suggests the use of specially shaped reference commands to suppress vibrations. Input shaping control introduced in [24] minimises vibrations during cart motion of the system. The closed loop control techniques rely on sensors to measure oscillations, but enhance robustness and are less sensitive to parameter changes. Among various closed loop techniques, [25] introduces a control approach that uses fuzzy logic with time delayed feedback for anti swing control in tower cranes. In [26], integral sliding mode control ensures precise tracking of desired positions while reducing oscillations in a system. In [27], a genetic algorithm based on Lyapunov stability theory is proposed for controlling underactuated systems. Most of these works primarily focus on mitigating undesirable swing, with limited attention being given to velocity control.

Further literature review has identified a few works focusing on MPC-PID hybrid combination for the control of underac-

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tuated systems. The works proposed in [28] disclose a method for controlling attitude control, momentum management, and stability of a spacecraft using an inner loop PID and an outer loop MPC. In [29], a method for controlling a multi degree of freedom flexible joint mechanical arm of a robot and stabilising the movement of the arm has been proposed. The method uses MPC as an upper level controller and the proportional derivative controller as a lower level controller, enhancing the stability of the robots. In [30], MPC and PID work in a loop to improve the system performance with PID gains tuned using the recursive least square method. The work in [31] proposes a cascaded MPC implementation, with closed loop system dynamics sampled using zero dynamics and Proportional Derivative (PD) control at each time step. In [32], the concept of Model Predictive Interaction Control (MPIC) approach for robotic interactions has been discussed. PD control is integrated into a cascaded framework to regulate contact dynamics.

The previously mentioned studies thus emphasise the applicability of combined control strategies across various complex systems. To validate such control approaches, low dimensional underactuated systems [6], [7], [13], [33], [34], [35] are often employed as benchmark models, due to their ability to represent real world control challenges in a more tractable form. From the several benchmarks stated, the inverted pendulum is important since it is a multi output underactuated system, characterised by fast dynamics, non minimum phase behaviour and nonlinear nature.

The key contributions of this article can be summarised in two parts. First, a dual control structure that combines the goodness of PD control and warm started MPC scheme is proposed. The primary objective addressed by this strategy is the suppression of pendulum swing and the positioning of the arm of the rotary inverted pendulum system. MPC deals with nonminimum phase system dynamics and permits the optimisation to be formulated as per the constraints, ensuring safety with minimal control effort. The LQR gain values, calculated in advance, are incorporated at the initial stage of the MPC to provide a warm start. Since the MPC strategy directly applies a torque to the system, it does not provide scope for reducing the oscillations. To combat this, the system's velocity calculated by the MPC scheme is given as a reference to an inner PD control loop. Thus, the PD control loop provides a bounded swing action. Secondly, a real time experimental study that compares the proposed warm started MPC-PD approach with a conventional cold started MPC has been performed. Both controllers are tested using identical MPC weighting matrices, and their performance is evaluated for a square reference trajectory. The results show that the proposed strategy delivers better performance for both trajectory tracking and stabilisation of the pendulum. In addition to this introduction, section II discusses the mathematical modelling of the pendulum system, and section III presents a theoretical background to the warm start approach with the MPC-PD design. The hardware results have been presented in section IV. Final findings are concluded in section V. This proposed dual control strategy does not just focus on the "performance" and "optimality" paradigms; it also explores control strategies in terms of "safety." This work also aligns with the sustainable development goals, focusing on industry and innovation. In the area of computing, the proposed control strategy utilises real time computational control algorithms, which are crucial for industrial automation. By enhancing predictive capabilities, the approach can benefit industries seeking effective solutions for robotic manipulators and smart manufacturing. In the area of electric energy, the ability of the proposed control strategy to anticipate actuator movements and minimise oscillations can be utilised to improve the sustainable energy infrastructure. In the area of electronics, the integration of PD and MPC enhances energy efficient motion control in robotics and smart electronic devices.

II. Mathematical Modelling of the Benchmark Problem

This work presents a warm start based dual control strategy designed to ensure the arm's trajectory tracking while maintaining the pendulum inverted. This strategy focuses on the application of a warm started MPC framework with PD to solve the control problem efficiently. The system employed for testing is the Qube Servo-2 rotary inverted pendulum setup developed by Quanser, depicted in Fig. 1(a). As shown in Fig. 1(b), it has two degrees of freedom, is nonlinear and exhibits dynamic behaviour and high instability. Inherent dynamics of a group of interconnected pendulums are analogous to those of interlinked robotic systems. The pendulum like motion of keel stabiliser, instrumental in governing the rolling dynamics of a ship, serves as a pertinent example. Moreover, the dynamics associated with the slosh phenomenon in a launch vehicle can be replicated using pendulum systems. Table I presents the numerical values of the system parameters utilised in this work. By Euler-Lagrangian equation-

TABLE I
NUMERICAL VALUES OF SYSTEM PARAMETERS

System parameter	Numerical value
Length of the arm (L_r)	0.0895 m
Moment of inertia of the arm (J_r)	2.28×10^{-4} kg m^2
Length of the pendulum (L_p)	0.129 m
Location of centre of mass of pendulum ($L_p/2$)	0.0645 m
Moment of inertia of the pendulum (J_p)	1.33×10^{-4} kg m^2
Pendulum mass (m_p)	0.024 kg
Frictional coefficient of arm (b_r)	1x10^{-3}
Frictional coefficient of pendulum (b_p)	5x10^{-5}
Motor's mechanical time constant (τ_m)	$20 \times 10^{-3}s$

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}_i} - \frac{\partial L(q,\dot{q})}{\partial q_i} = \tau_i \tag{1}$$

where τ_i is the torque at the external actuator.

Position of Centre of Mass (CoM) of the pendulum, with respect to x, y and z axes (X_x, X_y, X_z respectively) as shown in Fig. 1(b). are -

$$X_x = L_r cos\theta - 0.5 L_p sin\theta sin\alpha \tag{2}$$

$$X_y = L_r sin\theta + 0.5 L_p cos\theta sin\alpha \tag{3}$$

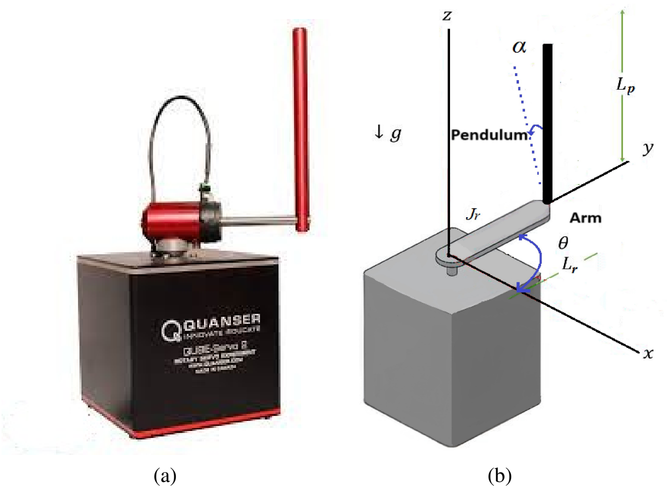

Fig. 1. (a) Qube Servo-2 platform (b) Free body diagram of a rotary inverted pendulum.

$$X_z = 0.5L_p cos\alpha \tag{4}$$

Substituting equations (2)-(4) in equation (1), the following equations of motion are obtained-

$$\begin{aligned}\ddot{\theta}(J_r + L_r^2 m_p + \frac{L_p^2}{4} m_p sin^2\alpha) + \dot{\alpha}\dot{\theta}\frac{L_p^2}{4} m_p sin(2\alpha) \\ -\dot{\alpha}^2 L_r \frac{L_p}{2} m_p sin\alpha + \ddot{\alpha}\frac{L_p}{2} L_r m_p cos\alpha = \tau - b_r\dot{\theta}\end{aligned} \tag{5}$$

$$\begin{aligned}\ddot{\theta} L_r \frac{L_p}{2} m_p cos\alpha - \dot{\theta}^2 \frac{L_p^2}{4} m_p sin(2\alpha) + \ddot{\alpha}(J_p + \frac{L_p^2}{4} m_p) \\ -m_p g \frac{L_p}{2} sin\alpha = -b_p\dot{\alpha}\end{aligned} \tag{6}$$

Here g represents gravity acceleration, θ and α represent the arm and pendulum positions respectively, and τ represents the torque directly acting on the arm. Since τ appears only in equation (5), the system is underactuated and difficult to control.

III. Proposed Controller

The proposed control problem consists of a dual layer structure. The inner loop uses a PD controller to achieve rapid anti sway and the outer loop uses warm started MPC to effectively solve the online constraint optimisation problem. Fig. 2 gives a detailed diagram of the dual control structure.

A. Tailored QP Problem involving Warm Start

MPC with a cold start fails to precisely solve the optimisation problem for an open loop, unstable system because of the absence of information regarding the initial condition. In this work, MPC is formulated as a Quadratic Programming (QP) problem and is initialised through a precomputed warm start. A control sequence based on LQ theory, which is close to the optimal solution is computed to warm start the MPC optimisation. By this method, the available computation time can be used to refine the system's input. This approach has

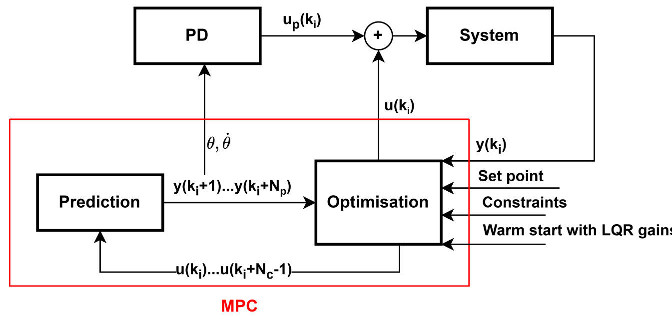

Fig. 2. Proposed warm start based MPC-PD dual control.

been developed with reference to the work quoted in [19], [36], [37] and [38].

In discrete time, cost function is designed to balance state regulation and control effort, ensuring system stability. It is given by-

$$J = \sum_{k_i=0}^{\infty} x^T(k_i)Qx(k_i) + u^T(k_i)\bar{R}u(k_i) \tag{7}$$

Here k_i denotes time, matrix Q decides on the rate at which the states $x(k_i)$ of the system converge, and $\bar{R}$ decides on the rate at which the input $u(k_i)$ gets actuated. In this work, the LQR weighting matrices Q and $\bar{R}$ have been designed using Bryson's rule [10], given as -

$$Q = diag(\tfrac{1}{x_{max}^2}) \text{ and } \bar{R} = (\tfrac{1}{u_{max}^2})$$

with x_{max} and u_{max} being the maximum permissible deviation on the states and control input, respectively. The discrete time Riccati equation [39] determines the optimal cost-to-go matrix P, which in turn defines the LQR gain K as -

$$K = (\bar{R} + B_{\bar{m}}^T P B_{\bar{m}})^{-1} B_{\bar{m}}^T P A_{\bar{m}} \tag{8}$$

where $A_{\bar{m}}$ and $B_{\bar{m}}$ are the state and input matrices of the system, respectively. The resulting control signal obtained is -

$$u_{lq}(k_i) = -Kx(k_i) \tag{9}$$

When constraints are inactive, MPC is initialised with a warm start of bounded control, generated using equation (8) [19]. Upon receiving the next update of $x(k_i)$, the predetermined starting point improves the control's convergence. Violation of the constraints initiates the MPC control law in a receding fashion. In the absence of constraints, the performance of MPC is equivalent to that of LQR when using a long prediction horizon. Based on this, the same set of weighting matrices, tuned based on Bryson's rule, is used for the conventional MPC and warm started MPC. These weights are then fine tuned while the control problem incorporates constraints.

B. Outer Loop Rate based MPC Design

MPC uses finite prediction (N_p) and control (N_c) horizons. A state space formulation of MPC is chosen for this work. Using this, the information required for predicting can be extracted from state variables. The MPC's warm start is infused with the system model to predict the system's future behaviour, following which control actions are generated by MPC and PD to bring the system to its desired state.

1) System Description: A discrete, linear time invariant system is represented in state space as-

$$x_{\bar{m}}(k_i+1) = A_{\bar{m}}x_{\bar{m}}(k_i) + B_{\bar{m}}u(k_i) \tag{10}$$

$$y(k_i) = C_{\bar{m}}x_{\bar{m}}(k_i) \tag{11}$$

where k_i denotes time, $\bar{m}$ denotes model, $u(k_i) \in \mathcal{U} \subset \mathbb{R}^m$ is the manipulated variable vector , $x_{\bar{m}}(k_i) \in \mathcal{X} \subset \mathbb{R}^n$ is the state variable vector, $y(k_i)$ is the output vector, $A_{\bar{m}}$ is a $n{\times}n$ state matrix, $B_{\bar{m}}$ is $n{\times}m$ input matrix and $C_{\bar{m}}$ is a $p{\times}n$ output matrix. Based on the rate based approach, the state space equations are remodelled as -

$$\Delta x_{\bar{m}}(k_i+1) = A_{\bar{m}}\Delta x_{\bar{m}}(k_i) + B_{\bar{m}}\Delta u(k_i) \tag{12}$$

$$y(k_i+1) - y(k_i) = C_{\bar{m}}A_{\bar{m}}\Delta x_{\bar{m}}(k_i) + C_{\bar{m}}B_{\bar{m}}\Delta u(k_i) \tag{13}$$

Equations (12) and (13) are combined to produce the following augmented state space model -

$$\underbrace{\begin{bmatrix}\Delta x_{\bar{m}}(k_i+1)\\ y(k_i+1)\end{bmatrix}}_{x(k_i+1)} = \underbrace{\begin{bmatrix}A_{\bar{m}} & 0^T\\ C_{\bar{m}}A_{\bar{m}} & I\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\Delta x_{\bar{m}}(k_i)\\ y(k_i)\end{bmatrix}}_{x(k_i)} + \underbrace{\begin{bmatrix}B_{\bar{m}}\\ B_{\bar{m}}C_{\bar{m}}\end{bmatrix}}_{B}\Delta u(k_i) \tag{14}$$

$$y(k_i) = \underbrace{\begin{bmatrix}0 & I\end{bmatrix}}_{C}\begin{bmatrix}\Delta x_{\bar{m}}(k_i)\\ y(k_i)\end{bmatrix} \tag{15}$$

where I is the identity matrix with dimensions $p \times p$ and A, B, C are the augmented state space matrices. The general equations of future state variables and predicted output variables are developed as the following equations -

$$\begin{aligned} x(k_i+N_p|k_i) &= A^{N_p}x(k_i) + A^{N_p-1}B\Delta u(k_i)\\ &+ ... + A^{N_p-N_c}B\Delta u(k_i+N_c-1)\end{aligned} \tag{16}$$

$$\begin{aligned} y(k_i+N_p|k_i) &= CA^{N_p}x(k_i) + CA^{N_p-1}B\Delta u(k_i) +\\ &... + CA^{N_p-N_c}B\Delta u(k_i+N_c-1)\end{aligned} \tag{17}$$

Using equations (16) and (17), the following is obtained-

$$Y = Fx(k_i) + \phi\Delta U \tag{18}$$

where

$$\Delta U = [\Delta u(k_i)\ \Delta u(k_i+1)\ ...\ \Delta u(k_i+N_c-1)]^T \tag{19}$$

$$Y = [y(k_i+1|k_i)\ y(k_i+2|k_i)\ ...\ y(k_i+N_p|k_i)]^T \tag{20}$$

wherein $\Delta u(k_i+j)$ is the future control movement defined in vector ΔU with j= 0,1, ..., $N_c - 1$ and $y(k_i+j|k_i)$ is the predicted system output defined in vector Y with j= 1, 2, ..., N_p and,

$$F = \begin{bmatrix}CA\\ CA^2\\ \vdots\\ CA^{N_p}\end{bmatrix}, \phi = \begin{bmatrix}CB & 0 & \cdots & 0\\ CAB & CB & \cdots & 0\\ CA^2B & CAB & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_c}B\end{bmatrix}$$

The cost function (7) is now redefined with integral action for set point tracking, based on rate based approach as -

$$\begin{aligned} J = (\bar{R}_s - Fx(k_i))^T(\bar{R}_s - Fx(k_i)) - 2\Delta U^T\phi^T(\bar{R}_s\\ -Fx(k_i)) + \Delta U^T(\phi^T\phi + \bar{R})\Delta U\end{aligned} \tag{21}$$

where $\bar{R}_s$ is the set point information. The control signal for the unconstrained system is thus obtained as -

$$\Delta U = (\phi^T\phi + \bar{R})^{-1}\phi^T(\bar{R}_s - Fx(k_i)) \tag{22}$$

This process is iterated, resulting in a receding horizon control law. In practical systems, inputs are always bounded. Ignoring control saturation can significantly degrade closed loop performance when constraints are active. Therefore, system input constraints need to be considered while minimising the cost function. The upper and lower bounds on the input amplitudes are defined as $u_{min} \leq u(k_i) \leq u_{max}$. However, since the system model is based on a rate based approach, equation (21) is formulated as a function of ΔU , while equation constraints are defined based on $u(k_i)$. To overcome saturation effects, $\Delta u(k_i)$ is calculated when the saturation is reached, and this information is incorporated to predict the state variables. Since the system has only one hard constraint, this modification was feasible. The samples of control input can be represented in matrix form as -

$$\begin{bmatrix}u(k_i)\\ \vdots\\ u(k_i+N_c-1)\end{bmatrix} = \underbrace{\begin{bmatrix}I\\ \vdots\\ I\end{bmatrix}}_{C_1}u(k_i-1) + \underbrace{\begin{bmatrix}I & \cdots & 0\\ \vdots & \ddots & \vdots\\ I & \cdots & 0\end{bmatrix}}_{C_2}\begin{bmatrix}\Delta u(k_i)\\ \vdots\\ \Delta u(k_i+N_c-1)\end{bmatrix} \tag{23}$$

Using equation (23), a transformation is thus applied to convert the bounds of $u(k_i)$ into the constraints on ΔU, resulting in the following matrix inequality -

$$M\Delta U \leq \gamma \tag{24}$$

where M=$\begin{bmatrix}-C_2\\ C_2\end{bmatrix}$, γ= $\begin{bmatrix}-u_{min} + C_1u(k_i-1)\\ u_{max} - C_1u(k_i-1)\end{bmatrix}$

Using the cost function (21) and the constraints (24), the optimisation problem can be converted into the following structure -

$$J = \frac{1}{2}\Delta U^T H\Delta U + \Delta U^T f \tag{25}$$

subject to

$$M\Delta U \leq \gamma$$

where H=2$\phi^T\phi$ and f=-2$\phi^T(\bar{R}_s - Fx(k_i))$.

C. Inner Loop PD with Velocity Control

The MPC strategy is designed to apply direct control to the arm, primarily focusing on position control and not specifically addressing the reduction of oscillations. To guide the pendulum to attain a steady state velocity, while minimising excessive oscillations, an inner loop PD is proposed. The arm velocity calculated by the outer loop MPC is provided as a reference for the derivative gain K_d while the arm velocity error serves as a reference for the proportional gain K_p, based on which PD controller generates $u_p(k_i)$. The gains have been tuned based on the root locus technique mentioned in [40]. In this setup, the control $u(k_i)$ generated by the MPC serves as a feedforward signal, as shown in Fig. 2. Predictions for the velocity reference at the next instant (k_i+1) are made by applying a combined control $u_p(k_i) + u(k_i)$. A detailed diagram illustrating the proposed warm start infused MPC-PD technique has been developed, as depicted in Fig. 3.

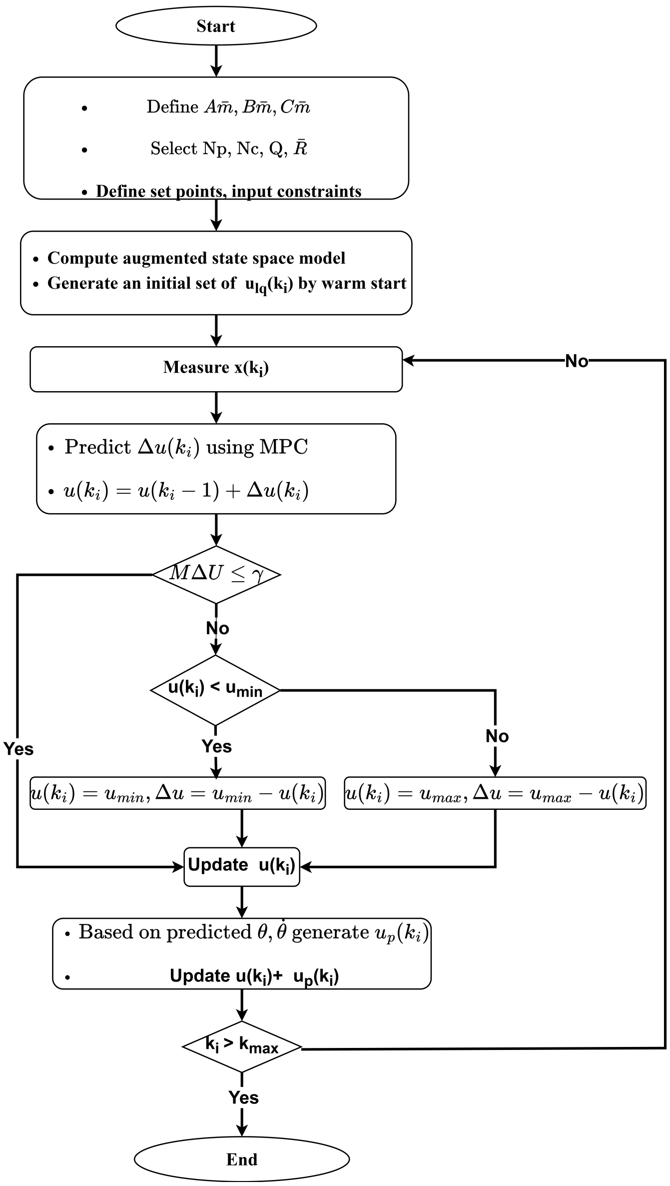

Fig. 3. Flow chart depicting the proposed warm started MPC-PD dual control strategy.

IV. Experimental Results obtained using Proposed Dual Control Strategy

The experiments performed in this section demonstrate that the proposed warm start based dual control structure ensures an anti sway action, under safe control. The tests have been conducted on a rotary inverted pendulum system Qube Servo-2 built by Quanser. The components of the rotary inverted pendulum platform and the experimental setup are -

- Computer
- Data Acquisition (DAQ) board
- E8P-512-118 optical encoder kit
- PWM voltage controlled power amplifier
- CL40 series coreless DC motor model 16705

The computer runs with a Windows 11 operating system, Matlab 2021a, which interacts with the DAQ through the Quarc target library. Following a systematic procedure, it was determined that the warm started MPC-PD exhibited stable operation on the hardware when operated with a sampling time equal to half the mechanical time constant $\frac{\tau_m}{2}$=10ms. The arm link was operated at a time varying reference of $\pm 15°$, and the weighting parameter for MPC was chosen as $\bar{R}$ = 0.2. For the inner PD control loop, the gain parameters were set to K_p = 2 and K_d = 0.4. The horizons were chosen as N_p =50, N_c=10 with control input constraints as $u_{min} = -10V$ and $u_{max} = 10V$. The experimental testing for a total time of 50s is presented here.

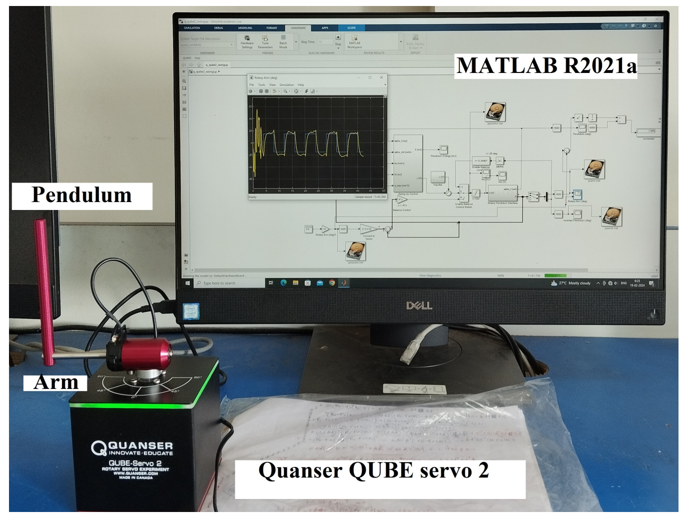

Fig. 4. Real time implementation of proposed warm start based MPC-PD on Quanser Qube Servo-2 platform.

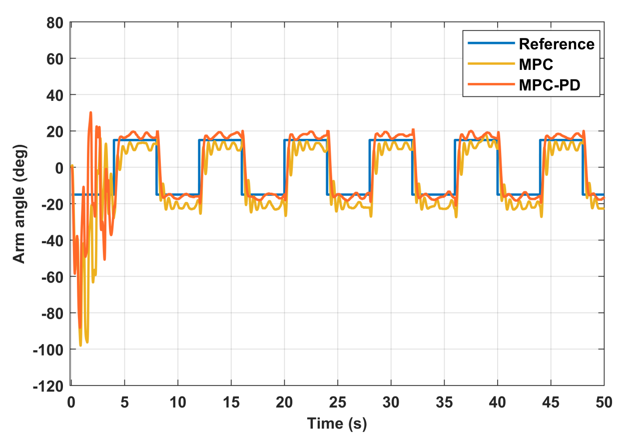

Fig. 5. Arm trajectory tracking by MPC (conventional) and MPC-PD (warm start based) techniques.

TABLE II
Comparison of System Performance based on Different Control Strategies

Performance indices	Conventional MPC	**Warm started MPC-PD**
ISE of arm trajectory	2.84	1.37
RMS of control voltage	0.83	0.69
Peak of control voltage (V)	4.9	2.8
Peak of pendulum deviation (deg)	7.7	2.2

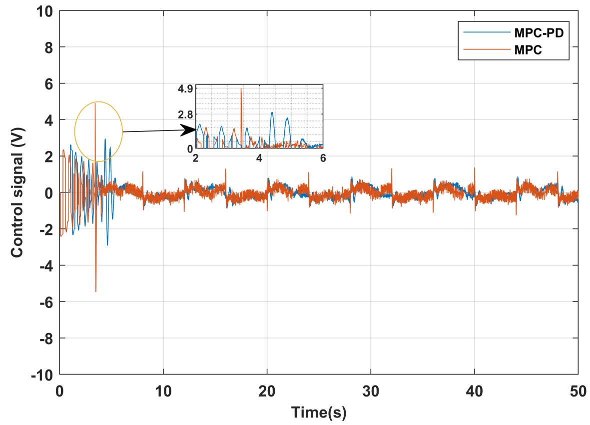

Fig. 6. Control signal generated using MPC (conventional) and MPC-PD (warm start based) techniques.

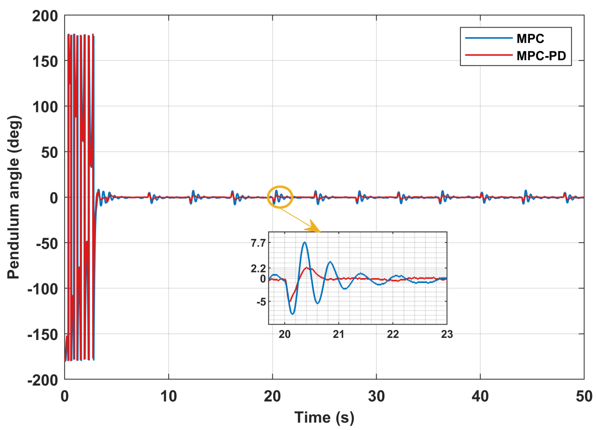

Fig. 7. Pendulum angle tracking by MPC (conventional) and MPC-PD (warm start based) techniques.

Fig. 4 presents the implementation of the proposed warm started MPC-PD dual control strategy on the hardware setup. Fig. 5-7 depicts a comparative analysis of the unstable system's performance under conventional MPC and warm started MPC-PD, respectively. As shown in Fig. 5, the warm started MPC-PD reduces oscillations around the setpoint. In terms of tracking performance, the Integral of Squared Error (ISE) for the arm is lower with the proposed control (1.37 vs. 2.84), demonstrating improved trajectory tracking. The proposed control significantly reduces peak control effort from 4.9V to 2.8V, as shown in Fig. 6, which helps in preventing actuator saturation and ensures smoother operation. The RMS of the control effort of warm started MPC-PD is lower (0.69 vs. 0.83), thereby being improved by 16.87%. This indicates lesser power consumption, which is critical for the actuator. Additionally, by using warm started MPC-PD the peak pendulum deviation is reduced from 7.7 to 2.2 degrees, as shown in Fig.7, confirming better stabilisation. The proposed warm started MPC-PD control achieves better overall performance by reducing the control signal peak, improving arm tracking, and suppressing pendulum oscillations more effectively. All the performance matrices have been tabulated in Table II.

V. Conclusion

The proposed warm start based MPC-PD dual control technique takes into account critical factors that are often overlooked by fixed value control techniques. The control problem involves the coordination of the pendulum's stabilisation alongside maintaining arm trajectory tracking. Warm started MPC addresses constraints, continuously recalculates the states, and correspondingly generates the control signal in real time. The inner loop PD aids in velocity control of the system and improves the performance. This experimental study focuses on creating a control input that optimises the system's inherent dynamics, effectively utilises nonlinear behaviour, and aids in trajectory tracking for achieving smooth stabilisation. The results presented in this work demonstrate that the system's behaviour remains bounded and converges to the desired state, without causing the actuator to saturate. The proposed scheme for the rotary inverted pendulum can be extended to other underactuated systems like cart pole system, acrobot, flexible arm joint and crane since they share common challenges like unstable equilibrium points, limited control inputs, and complex dynamics. The warm started MPC component can ensure constraint satisfaction, while the PD component can enhance response speed, ensuring rapid corrective actions before the next state update.

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