A Novel Approach to Performance Evaluation of Current Controllers in Power Converters and Electric Drives Using Non-Parametric Analysis

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***Abstract*—In the field of current controllers for power converters and electric motor units, conventional figures of merit (FMs) such as mean squared error (MSE), integral of time multiplied by absolute error (ITAE), or total harmonic distortion (THD) have traditionally been used. The stochastic nature of these FMs introduces variability, which often results in inaccurate comparisons of the controllers' performance. A parametric statistical methodology has recently been proposed to address this issue. However, it presents certain limitations, such as the assumption of normality. In response to this, the adoption of a non-parametric methodology is suggested in this work, which promises a precise evaluation of the efficiency of current controllers. In this study, we demonstrate that, under the parametric approach, when the assumption of normality is violated, there is a significant increase in Type I error. Furthermore, we show that the non-parametric Mann-Whitney U test offers greater sensitivity compared to its parametric counterpart under these circumstances. Thus, the proposed methodology aims to optimize the decision-making process in designing high-performance current controllers for power converters and electric motor applications. This allows for design decisions grounded in rigorous and statistically-based evaluations. The effectiveness of this methodology is confirmed through its application to a real dataset, enhancing its practicality and contributing to a deeper understanding of the subject matter.**

Link to graphical and video abstracts, and to code: https://latamt.ieeer9.org/index.php/transactions/article/view/9139

***Index Terms*—current controllers, power converters, electric motors, decision-making, non-parametric methodology.**

I. Introduction

IMPROVING control techniques and innovating with new power converters and electric motor drive methodologies have become a focal point in recent research [1].

These emerging technologies and methodologies are crucial for applications across the sciences and engineering fields, ranging from the analysis of new electric vehicle systems to wind energy conversion and distributed generation systems [2], [3]. This continuous drive for superior control and monitory process strategies is fueling research in this area.

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TABLE I
Comparison with Some Reported Figures of Merits used to Evaluate Current Controllers

Application	Figure of Merit	Reference
3-phase IM	MAE, RMRE	[4]
Wind Energy	THD	[5]
Six-phase IM	THD, Switching losses	[6]

Traditionally, assessment methods have relied on graphical comparison signals from oscilloscopes. However, recent advancements favour the adoption of figures of merit (FMs), which evaluate measured samples from current sensors (e.g. Hall effect sensors) against the desired current, such as mean squared error. This exposes the stochastic nature of these metrics, and prompts the need for robust evaluation criteria.

When using FMs to evaluate control strategies, it is essential to consider that the accuracy and precision of the sensors used in the experiments can have a significant impact. For instance, Hall effect sensors, commonly utilized for their cost-effectiveness and simplicity, may introduce errors due to offset drift, temperature sensitivity, and limited resolution. These inaccuracies can affect the reliability of the measurements and subsequently influence the calculated FMs and the overall interpretation of the control strategy's performance. Consequently, it's essential to recognize these limitations when interpreting experimental results, as they can impact the comparability and generalizability of the findings. Table I summarizes recent works using FMs to compare current control strategies, highlighting the persistent quest for high-performance closed-loop current control strategies in the existing literature [7], [8].

The comparison of groups using statistical methods is a widely accepted approach [9], [10]. This study builds upon previous research [11] to enhance the performance evaluation of power converters and electric motor drives. It introduces scientific, statistical criteria to establish a precise method for comparing current controllers in power converters and electric drives. In previous work, we focused on parametric statistical methods, assuming that the underlying data follows a specific probability distribution with fixed parameters. However, for the hypothesis test to be valid, the distribution from which the samples are drawn follows a normal distribution [12], [13]. This assumption is not always met in real-world scenarios, leading to recent proposals for using non-parametric tests in different fields [14]– [18].

Several studies have applied non-parametric methods with successful results. In [19] compares parametric and non-parametric approaches for determining probability density functions of load flow outputs. The unscented transform and two-point estimation approaches were used for parametric methods, while saddle point approximation and kernel density estimation were employed for non-parametric methods. The study evaluates these methods regarding accuracy and execution time across several IEEE test systems, concluding that non-parametric methods provide reliable results and can significantly reduce computational time.

Also, in [20], they compare various modelling approaches for predicting freight generation, focusing on the performance of parametric and non-parametric methods. Using data from 432 establishments in Kerala, India, the results show that non-parametric support vector regression consistently outperforms parametric models. Multiple classification analysis performs better for suburban models, while robust regression performs well in some industrial segments. However, non-parametric models demonstrate superior predictive accuracy compared to parametric approaches.

This type of comparison reinforces the need to consider non-parametric methods in uncertainty analysis, especially when the assumptions of normality are not met, and could serve as a basis for justifying non-parametric approaches in broader control systems.

The assumption of normality, commonly applied in parametric analysis, is not always valid for stochastic metrics such as mean squared error (MSE) or total harmonic distortion (THD). When the distribution of these metrics deviates significantly from normality, as often occurs in practical applications due to uncontrolled disturbances and noise in power systems, the conclusions derived from parametric analysis can be misleading. This can lead to inaccurate assessments of the performance of current controllers. In practice, these incorrect evaluations can result in the adoption of controller designs that, while appearing optimal under parametric metrics, are not in real operating conditions. This is particularly relevant in industrial applications where operating conditions vary significantly, and stochastic behaviour can greatly affect controller performance. Consequently, design decisions based on such evaluations may not meet industrial settings' efficiency or reliability requirements.

By adopting a non-parametric methodology such as the Mann-Whitney U test, these issues are mitigated, as it does not require assumptions about the data distribution. This allows for a more robust and accurate controller performance evaluation, leading to better-informed design decisions and more efficient and reliable current controllers. Using non-parametric approaches helps avoid the need to make specific assumptions about the underlying distribution. This is beneficial in situations where the nature of the data is uncertain or complex. Within this framework, this paper will primarily explore the following questions:

1) How should we measure the performance of current controllers when their performance does not follow to a normal distribution?
2) To what degree are comparisons less accurate when we assume normality but is not met?

Hence, this study aims to be the first to use non-parametric statistical techniques to allow for a detailed comparison of two current control methods. As a result, the main contribution of this research is to expand the use of reliable statistical methods, making it possible to compare two current control techniques based on FMs even in scenarios where the normality assumption is not met.

The rest of this paper is organized as follows. Section II provides an overview of the parametric comparison procedure. In Section III, we evaluate the accuracy of this comparison under the assumption of normality and address scenarios where this assumption might not hold. The theoretical framework guiding the proposal of an appropriate approach for comparing the performance of current controllers when their data deviates from a normal distribution is presented in Section IV. Section V applies this approach to real data, making the practical implementation of the proposed method more accessible. Finally, Section VI provides conclusive insights.

II. Parametric Approach based on T-Test

This section summarizes the parametric comparison and introduces the statistical principles that will be used in the following sections. The proposal is primarily based on the guidelines for Type A evaluation of standard uncertainty [21].

Let $X_1, X_2, ..., X_n$ be a sequence of independent and identically distributed random variables of a probability distribution function (PDF) having a of mean μ and a non-zero variance σ^2. Then, as $n \to \infty$, the parametric approach is based on the Central Limit Theorem (CLT) [22].

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{1}$$

The random variable $\overline{X}$ converges to a normal distribution with a mean represented by:

$$\mu_{\overline{x}} = \mu, \tag{2}$$

while the variance is

$$\sigma_{\overline{x}}^2 = \frac{\sigma^2}{n}. \tag{3}$$

By applying the CLT, the bilateral confidence interval (CI) with $1-\alpha$ confidence level (CL) for $\mu_{\overline{x}}$ is:

$$\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}}) = \overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{4}$$

being $z_{\alpha/2}$ the percentile $\alpha/2$ of the standard normal distribution and $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

In real applications like current control techniques for power electronic converters or electric motor drives, variance σ^2 is unknown. Then, in this situation, if $X \sim \mathcal{N}(\mu_x, \sigma)$ even for small n ($n < 30$) the way of applied (4) providing better results is what follows:

$$\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}}) = \overline{x} \pm t_{\alpha/2,n-1}\frac{\hat{S}_x}{\sqrt{n}} \quad (5)$$

where $t_{\alpha/2,n-1}$ is the $\alpha/2$ percentile of a t-distribution with $n-1$ degrees of freedom and $\hat{S}_x$ is an estimator of σ, defined by

$$\hat{S}_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}. \quad (6)$$

Let's consider two random variables, X and Y, representing the same FM for two current control methods, A and B. In general, we assume an unknown PDF. Without loss of generality, let's suppose that they have meant μ_x and μ_y, as well as variances σ_x^2 and σ_y^2, respectively. The t-test compares these two current controllers from a parametric perspective. In this context, we use the estimation of the CI, considering the difference in means, $\delta = \mu_x - \mu_y$.

The comparison between controllers, employing the parametric t-test, begins with the assumption that both controllers are equally efficient, referred to as the null hypothesis, denoted as H_o. On the contrary, the alternative hypothesis, denoted as H_1, represents the hypothesis the researcher seeks to confirm or reject. These hypotheses are as follows:

$$H_0 : \delta = 0$$

vs.

$$H_1 : \delta \neq 0$$

Null hypothesis H_o assumes that both controllers (A and B) are equally efficient based on some FMs, and H_1 establishes the superiority of one of them. If the $0 \in \mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}})$, there is not enough statistical evidence to reject the null hypothesis. Otherwise, accept the H_1.

In statistical hypothesis testing, it's recognized that errors can occur when making decisions based on random samples. However, the CLT helps control these errors from a probabilistic perspective. Table II provides an overview of the types of errors to consider.

If both controllers are equally efficient but applying a t-test to experimental samples incorrectly suggests a difference, a Type I error is committed. The probability of this error is represented by α, which is complementary to the CL.

On the other hand, a Type II error, represented by β, occurs when one controller is superior to the other, but the t-test fails to detect this difference.

The power of the test measures the test's ability to reject a null hypothesis correctly when it is false. In probabilistic terms, it's represented as the complement of β, or $1 - \beta$.

III. Non-normality Impact

The cost implications of the increased Type I error are explained next to fulfil one of the objectives of this article, namely to assess the accuracy of the comparison when assuming normality even when this assumption is not met.

TABLE II
Type I and Type II Errors

Decision based on sample	Real situation $\delta \neq 0$ (H_0 false)	$\delta = 0$ (H_0 true)
$\delta \neq 0$	No error	Type I error α
$\delta = 0$	Type II error β	No error

One of the primary reasons why researchers avoid the use of the t-test is the lack of normality in the sample distributions. While the CLT does not necessarily require normal distributions, convergence is slower when normality is absent. To illustrate this, we simulated two probability distributions with identical means. One follows a normal distribution, while the other is asymmetric, specifically following a log-normal distribution (refer to Fig. 1).

For various sample sizes ranging from 10 to 600, 1000 random samples are drawn from each probability distribution, and the means are compared using the t-test. The percentage of tests that yield significance at a 5% level is determined. Ideally, the rejection rate should closely match the nominal 5%. However, as depicted in Fig. 2, for sample sizes below 100, the type I error significantly exceeds the nominal level. Only for substantial sample sizes, beginning at 200, does the type I error begin to approach the nominal value.

When the normality of the distribution cannot be assumed, we recommend using the Mann-Whitney U test, also known as the Wilcoxon rank sum test. This test is suitable for assessing differences between two groups on a single, ordinal variable with no specific distribution [24], [25].

The previous experimentation showed the impact of non-normality on the type I error. An experiment was performed to see the impact in terms of power. For this, samples with sizes 5 to 50 were simulated. Two possible scenarios were considered: one where one of the samples does not comply with the assumption of normality and another where both do not comply. Likewise, the parametric t-test and the nonparametric Mann-Whitney U test were applied. As shown in Figs. 3-4, the Mann-Whitney U test is robust for both cases. While the t-test lacks statistical power when the samples are less than 50 and the assumption of normality is not met (see Fig. 3).

The Mann-Whitney U test was chosen as it is a robust non-parametric alternative to the t-test, which does not assume normality. This is crucial in our study, as our simulation

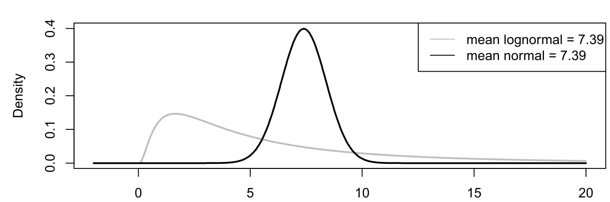

Fig. 1. Lognormal and normal distributions with the same mean.

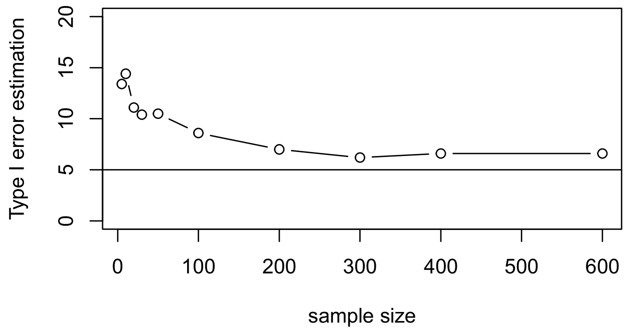

Fig. 2. Type I error estimation for different sample sizes under non-normality.

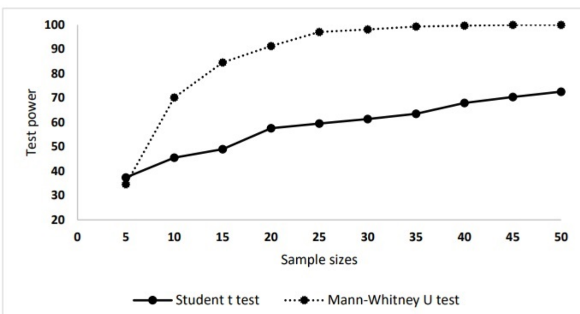

Fig. 3. Power of parametric and non-parametric tests when only one has a normal distribution.

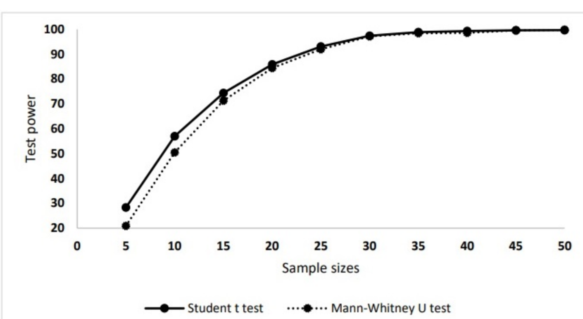

Fig. 4. Power of parametric and non-parametric tests when both have a normal distribution.

demonstrated that when the normality assumption is violated, and a parametric test is applied, it increases the likelihood of incorrectly selecting one controller over another, even when both have the same efficiency. This increase in Type I error, particularly for sample sizes below 200, highlights the limitations of parametric methods in these contexts.

Moreover, the power of the statistical test—defined as the probability of correctly detecting that one controller is better than another when it truly is—was compared between the t-test and the Mann-Whitney U test. The results showed that the U test had greater power for sample sizes smaller than 50, making it more effective in detecting real differences between controllers under small sample conditions or non-normal distributions, as is often the case in practical applications involving current controllers for power converters and electric motor units.

In the application of real data (section V), we used the Mann-Whitney U test to compare performance metrics (such as MSE and THD) between different controllers. This ensured that the comparisons accurately reflected the actual performance differences without being affected by the lack of normality in the data. As a result, we could identify which controllers offered more robust performance under real-world conditions, providing a more reliable basis for decision-making in current controller design.

A. Computational Complexity Comparison

The computational performance of the parametric t-test and the non-parametric Mann-Whitney U test differs slightly in theory. The t-test, with a computational complexity of $\mathcal{O}(n)$, involves straightforward calculations of means and variances. In contrast, the Mann-Whitney U test requires sorting the data and calculating ranks, leading to a slightly higher complexity of $\mathcal{O}(n \log n)$.

However, in practical applications, particularly in current control systems where sample sizes are generally below 100, the difference in computational complexity between these two methods is negligible. For small sample sizes (fewer than 1,000 observations), both tests converge quickly, with the Mann-Whitney U test only taking marginally longer than the t-test.

In this study, the computational time for both tests was measured in milliseconds, meaning there is no practical impact on the overall analysis. The main distinction between the two methods lies in their assumptions and statistical power. The t-test requires normally distributed data, while the Mann-Whitney U test does not, providing greater robustness in cases of non-normally distributed data.

Given the small sample sizes typically used in current control applications, the Mann-Whitney U test remains a computationally feasible and statistically robust alternative when the normality assumption is violated without introducing significant additional computational cost.

IV. Proposed Non-parametric Comparison Method

We present the theoretical framework to accomplish another significant contribution of this article: proposing the appropriate approach to comparing the performance of current controllers when their data do not follow a normal distribution.

As demonstrated in the previous section, the limitations of applying parametric tests due to non-normality can be addressed by significantly increasing the sample sizes.

Fig. 5 displays simulated random results from two controllers, A and B. Specific values from controller A are lower than certain values from controller B. If, by coincidence, these were the sole values collected, it could lead to erroneous conclusions. Evaluating them based on the mean would apply solely to symmetric probability distributions. Furthermore, using the t-test would be appropriate only if both distributions followed a normal distribution [11].

Let X denote the FM for controller A, and Y denote the FM for controller B. We have n_1 measurements for controller A and n_2 for controller B. It is a valid assumption that these measurements are independent, as both controllers are unrelated, and our objective is to compare them. As a case study, this paper will focus on two FMs: the MSE and the THD. The MSE between the reference and the measured stator currents in the (α-β) sub-spaces is performed by using the following equation

$$\text{MSE}(i_{\sigma s}) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(i_{\sigma s} - i^*_{\sigma s})^2} \tag{7}$$

where n represents the number of experiment replications, $i^*_{\sigma s}$ stands for the reference stator current, and $i_{\sigma s}$ represents the measured stator current, considering that σ belongs to the set $\{\alpha, \beta\}$. While the THD is obtained as follows:

$$\text{THD}(i_s) = \sqrt{\frac{1}{i^2_{s1}}\sum_{k=2}^{n}(i_{sk})^2} \tag{8}$$

where i_{s1} corresponds to the fundamental stator current and i_{sk} is the harmonic stator current (multiple of the fundamental stator current).

The performance of a current control method was considered better when its FM values were closer to zero, as indicated by the definitions in (7) and (8). While this theoretical value was challenging to use in practice, a practical comparison is made by evaluating whether the FMs of one current control method are, as a whole, lower than those of another. Considering this concept, when the two samples being compared come from the same population (i.e., they are equally efficient), all the observations when combined and sorted from smallest to largest, would be expected to be randomly interspersed, as depicted in Fig. 5. In contrast, if one of the samples comes from a population with values significantly greater or less than the other population, the observations will tend to group in such a way that those of one sample are predominantly above those of the other, as shown in Fig. 6. The statistical hypothesis testing based on this idea, using the Mann-Whitney U test, is as follows:

$$H_0 : P(X > Y) = P(X < Y) = 0.5 \tag{9}$$

vs.

$$H_0 : P(X > Y) > 0.5$$

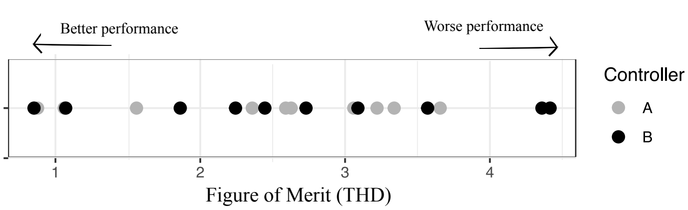

Fig. 5. Different controllers with equal efficiency.

Comparing the performance of current controllers, this test helps contrast the probability that an FM value of one type of controller A (X) surpasses a value of the same FM of another controller B (Y), with the probability that a value from B exceeds a value from A. Essentially, it checks if the values from one population tend to be stochastically larger than those of another. In this scenario, the alternative hypothesis H_1 suggests that the values of controller A exceed those of controller B. If this hypothesis is accepted, it implies that controller B is more efficient than controller A.

There are a few non-parametric methods available for comparing two groups:

- Median Test: This test compares the medians of two independent groups and is a straightforward alternative when comparing central tendencies. However, it is generally less powerful than the Mann-Whitney U test for detecting differences between distributions, especially when the sample sizes are small or there is a significant overlap in the distributions [26].
- Wilcoxon Signed-Rank Test: This test is suitable for comparing two related or paired samples. While effective for dependent data, it does not apply to our study, which focuses on comparing independent groups.
- Kolmogorov-Smirnov Test: This test compares the distribution of two independent samples rather than just central tendencies. It is useful when the interest lies in comparing the shape of the distributions, but in our case, the primary focus is on comparing central tendencies or rankings.

The Mann-Whitney U test was selected because it is specifically designed for comparing two independent groups without requiring the assumption of normality. Unlike the median test, it has greater power to detect distribution differences. Additionally, it is more appropriate than the Wilcoxon Signed-Rank test, as we are dealing with independent groups, and it focuses on rank-based comparisons rather than entire distribution shapes, making it more suitable than the Kolmogorov-Smirnov test for our purposes. Our simulation results further confirmed that the Mann-Whitney U test provides reliable sensitivity and accuracy, making it the most appropriate choice for our analysis.

The algorithm for calculating the Mann-Whitney U statistic for independent samples X and Y of sizes n_1 and n_2, respectively, is as follows [27]:

1) Create a vector of observations by combining samples and arranging them from smallest to largest.
2) Let $N = n_1 + n_2$ be the size of this combined vector, and assign a rank from 1 to N to each position in the joint sample.

Fig. 6. Controller B better than Controller A.

3) Sum the ranks for all observations in X to obtain R_X, analogously R_y
4) Calculate $U_X = n_1 \cdot n_2 + \frac{n_1(n_1+1)}{2} - R_X$
5) Calculate $U_Y = n_1 \cdot n_2 + \frac{n_2(n_2+1)}{2} - R_Y$

The value of the U statistic is the smaller of the statistics U_X and U_Y.

$$U = \min(U_X, U_Y)$$

The Mann-Whitney U distribution is a discrete distribution that depends on sample sizes. The U distribution is discrete for small samples, and critical value tables should be used. The U statistic approximates a normal distribution for large samples (usually with more than 20 observations in each sample). A normal approximation can calculate critical values and determine statistical significance. The normal approximation is based on the CLT and allows for more straightforward calculations. This is the approach taken in this article, with the condition that the sample sizes for both groups being compared are greater than 20. The approximation to the normal distribution, denoted as z, when we have sufficiently large samples, can be calculated using the expression:

$$z = \frac{(U - \mu_U)}{\sigma_U}$$

where μ_U and σ_U are the mean and standard deviation of U if the null hypothesis is true, and the following formulas give them:

$$\mu_U = \frac{n_1 \cdot n_2}{2}$$

and

$$\sigma_U = \sqrt{\frac{n_1 \cdot n_2 \cdot (n_1 + n_2 + 1)}{12}}$$

The methodological summary is presented in Fig. 7. The process begins with checking whether the samples originate from a normal distribution. In the case where the normality assumption is not rejected, a parametric approach is followed. When there is no significant difference between the variances, the t-test is applied; otherwise, the Welch correction is implemented. In the context of the methodology proposed in this article, if normality is rejected in either of the two groups to be compared, the non-parametric approach is chosen through the Mann-Whitney U test.

The open-source software R [28] implements the test (the Wilcoxon two-sample test) using the ‘wilcox.test‘ function. In the following section, we demonstrate its application to a real dataset.

To further clarify the implementation of the Mann-Whitney U test, Fig. 8 is presented. It illustrates the steps followed in the analysis to compare the performance of Controller A and Controller B using the Mann-Whitney U test. The process begins with the calculation of figures of merit, such as THD and MSE, which are then saved to an Excel file for further analysis.

Next, the data is imported into R, where the vectors corresponding to the figures of merit for both controllers are extracted. These vectors are then compared using the Mann-Whitney U test, which is employed to assess whether Controller A has a statistically significant lower central value than Controller B.

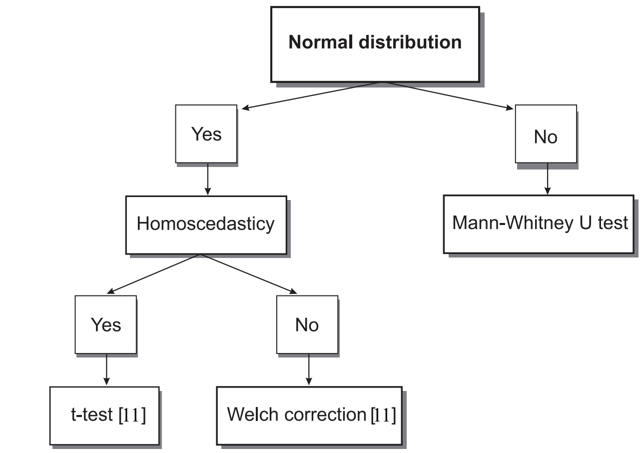

Fig. 7. Efficiency comparison evaluation flow.

The final decision is based on the p-value obtained from the test. If the p-value is less than 0.05, the null hypothesis is rejected, concluding that Controller A performs better than Controller B. Conversely, if the p-value is greater than or equal to 0.05, there is insufficient evidence to determine the superiority of one controller over the other.

This flowchart simplifies the specific methodology, providing a visual guide to the steps followed in the analysis and outlining how the decision-making process is structured based on the test results.

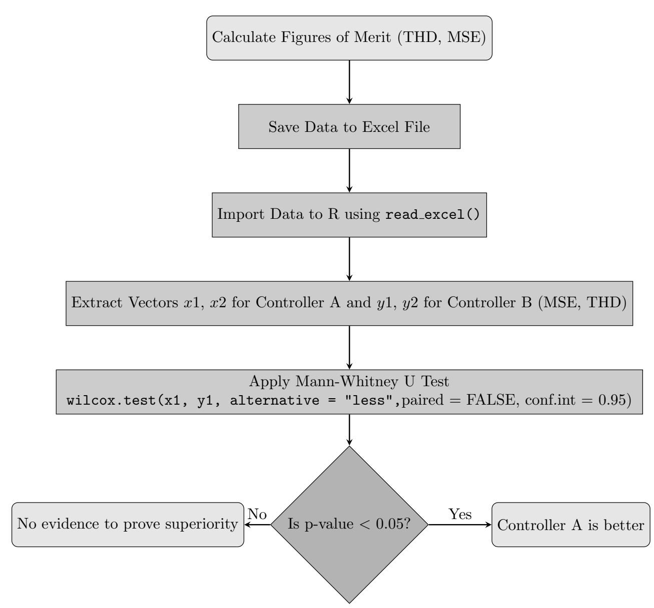

Fig. 8. Visual guide to the analysis of the Mann-Whitney U test for controller comparison.

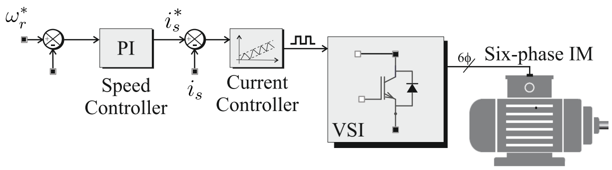

Fig. 9. Schematic diagram of the field-oriented control for a six-phase IM.

V. Real data application

A. Multiphase machine description and control

The system that will be used as a showcase is a multiphase machine (more than 3 phases). This machine can continuously operate even if one or more phases are open or with faults [29]. Moreover, the possibility to split the power into more phases allows the use of reduced components and leads to a more efficient use of the cable cross-sectional area [30]. All these advantages have motivated industrial applications of multiphase machines in propulsion systems [31], wind energy conversion systems [32], among others. However, a higher number of phases leads to control challenges.

Advanced control strategies of multiphase machines have been a leading research topic for more than a decade [33]. Most of the control approaches are extensions of well-known techniques for conventional three-phase machines such as direct torque control [34] or field-oriented control [35], and non-linear controllers like sliding mode control [36] and MPC [37], [38]. Extending the aforementioned control techniques to the post-fault operation has also been proposed [39]. Fig. 9 shows the field-oriented control technique applied to a multiphase machine, which consists of a proportional-integral (PI) outer speed control with an inner current controller. This schematic is considered in this paper with MPC as a current controller.

The most popular type of multiphase machine, namely the asymmetrical six-phase induction machine (IM), is considered in this paper. This machine is driven by a 2-level voltage source inverter (VSI). By using first-order Euler approximation, the discrete form representation of the state variable can be expressed in the form:

$$\begin{bmatrix} i_{s\alpha[k+1]} \\ i_{s\beta[k+1]} \\ i_{sx[k+1]} \\ i_{sy[k+1]} \\ i_{r\alpha[k+1]} \\ i_{r\beta[k+1]} \end{bmatrix} = \mathbb{A}. \begin{bmatrix} i_{s\alpha[k]} \\ i_{s\beta[k]} \\ i_{sx[k]} \\ i_{sy[k]} \\ i_{r\alpha[k]} \\ i_{r\beta[k]} \end{bmatrix} + \mathbb{B}. \begin{bmatrix} v_{s\alpha[k]} \\ v_{s\beta[k]} \\ v_{sx[k]} \\ v_{sy[k]} \end{bmatrix} \quad (10)$$

where $i_{s\alpha}$, $i_{s\beta}$ are the $\alpha-\beta$ stator current, i_{sx}, i_{sy} are the $x-y$ stator current and $i_{r\alpha}$, $i_{r\beta}$ represent the unmeasurable $\alpha-\beta$ rotor currents. The input voltages are denoted by $v_{s\alpha}$, $v_{s\beta}$, v_{sx}, v_{sy} and finally, $\mathbb{A}$ and $\mathbb{B}$ are defined in [36].

MPC is a technique based on the system model. For power converters, MPC takes advantage of the discrete nature of the model. A cost function shown below in (11) defines the desired behaviour, such as current tracking. For a six-phase IM, the cost function is typically evaluated 49 times. Then, the voltage vector that minimizes the cost function is selected and applied to the six-phase machine through the VSI during the next sample time.

$$J = |i^*_{s\alpha\beta[k+1]} - \hat{i}_{s\alpha\beta[k+1]}|^2 + \lambda_{xy}.|i^*_{sxy[k+1]} - \hat{i}_{sxy[k+1]}|^2. \quad (11)$$

Readers are referred to [40] for more details regarding MPC applied to power converters and drives, and [37], [41] for MPC applied to multiphase machines.

The second current controller used in this paper is the modulated model predictive control (M2PC) proposed in [42]. The M2PC uses a modulation stage based on a switching pattern to generate a fixed switching frequency. The duty cycles are generated using two active vectors and a null vector, which are applied to the converter using a given switching pattern.

B. Nonparametric comparison

The procedure for comparing the two controllers, MPC [37] and M2PC [42], will be outlined below. A six-phase IM supplied by a commercial VSI is used for that purpose. A constant voltage V_{dc} was sourced from a dc power supply system. The current controllers were implemented on the dSPACE MABXII DS1401 rapid prototyping platform. The results obtained were recorded and processed using a MATLAB/Simulink script.

Experimental tests were conducted using LA 55-P current sensors, with a frequency bandwidth ranging from direct current (dc) up to 200 kHz. Subsequently, the current measurements were converted to digital format using a 16-bit analog-to-digital converter. The position of the six-phase IM was determined using a 1024-pulses-per-revolution incremental encoder for rotor speed estimation. Additionally, a 5 HP (3.7285 kW) eddy current brake was employed to introduce variable mechanical loading to the system.

The performance of both controllers was evaluated using MSE and THD as FMs. Based on a total of 400 experimental tests, obtained for 200 tests for each MPC [37] and M2PC [42] current controllers, under the same condition: 16 kHz sampling time, 1500 rpm rotor speed, $i_q = 1$ A and $i_d = 1$ A. Applying (7)-(8), the following FMs are computed: in α sub-space. Basic statistics results are presented in Table III.

TABLE III
DESCRIPTIVE STATISTICAL PARAMETERS FOR THE FMs MSE_α AND THD_α, BASED ON 200 EXPERIMENTAL TEST FOR EACH MPC AND M2PC CONTROLLERS

Descriptive statistical	MSE_α		THD_α	
parameters	MPC	M2PC	MPC	M2PC
Minimum	0.0615	0.0933	5.9077	7.7733
Maximum	0.0678	0.1087	9.2271	9.7536
Mean	0.0654	0.1019	7.3758	9.0078
SD	0.0011	0.0030	0.8082	0.3428

Normality test

Random samples equal to $\{n_1, n_2\}$ = 30 were taken in each group of 200 samples obtained to compare both controllers. To determine whether the parametric or non-parametric method should be employed for comparing the performance of both current controllers, a normality test should be conducted [22]. The null hypothesis (H_0) and the alternative hypothesis (H_1) are established as follows:

H_0 : The FM (i.e. the MSE$_\alpha$) has a normal distribution for both controllers (MPC and M2PC).

H_1 : The FM (i.e. the MSE$_\alpha$) does not have a normal distribution for both controllers (MPC and M2PC).

A significance level of $\alpha = 0.05$ is used, meaning the null hypothesis (H_0) will not be rejected if the p-values exceed 0.05.

The vectors x_1 and y_1 contain the MSE$_\alpha$ data for the MPC and M2PC controller, respectively. For the THD$_\alpha$, x_2 and y_2 are defined analogously.

To determine normality, the Kolmogorov-Smirnov test with the Lilliefors correction is applied using the following code in R [28]:

```
library(nortest)
lillie.test(x1)
lillie.test(y1)
lillie.test(x2)
lillie.test(y2)
```

Based on the results presented in Table IV, the THD$_\alpha$ do not conform to the normality assumption in the MPC controller. Therefore, the parametric method cannot be employed to compare these FM. In this case, conducting a non-parametric test, as described in this study, is imperative.

TABLE IV
NORMALITY TEST

FM	Controller	p-value	Can we assume normal distribution?
MSE$_\alpha$	MPC	0.8357	Yes
MSE$_\alpha$	M2PC	0.1815	Yes
THD$_\alpha$	MPC	0.0105	No
THD$_\alpha$	M2PC	0.7218	Yes

Non-parametric comparison

Table V presents the measured THD$_\alpha$ values for each controller. Upon arranging the observations from lowest to highest, the first 25 values belong to the MPC. The lowest value for the M2PC appears at position 26. The maximum value for the MPC is located at position 50. To conclude whether this evidence is sufficient to determine that the MPC outperforms the M2PC, we conduct the Mann-Whitney U test, starting with establishing the null and alternative hypotheses.

$H_0 : P(x_2 > y_2) = P(y_2 > x_2)$, that is, base on the THD$_\alpha$ no controller is more efficient than the other.

H_1 : Base on the THD$_\alpha$, MPC controller is more efficient than M2PC.

The code to perform the Mann-Whitney U test using the free software R [28] is as follows:

```
wilcox.test(x2, y2, alternative = "less",
paired = FALSE)
```

The code compares the vector x_2 with the vector y_2. The alternative hypothesis should be chosen as “less”, and since

TABLE V
RANK ASSIGNMENT BASED ON THD VALUES

THD$_\alpha$	Controller	Rank	THD$_\alpha$	Controller	Rank
6.32	MPC	1	8.58	M2PC	31
6.36	MPC	2	8.63	M2PC	32
6.44	MPC	3	8.67	M2PC	33
6.47	MPC	4	8.70	M2PC	34
6.49	MPC	5	8.75	MPC	35
6.50	MPC	6	8.76	MPC	36
6.51	MPC	7	8.79	M2PC	37
6.52	MPC	8	8.83	M2PC	38
6.58	MPC	9	8.84	M2PC	39
6.63	MPC	10	8.86	M2PC	40
6.68	MPC	11	8.86	MPC	41
6.70	MPC	12	8.93	M2PC	42
6.77	MPC	13	8.94	M2PC	43
6.82	MPC	14	8.94	M2PC	44
6.94	MPC	15	8.95	M2PC	45
7.22	MPC	16	8.97	M2PC	46
7.42	MPC	17	9.01	M2PC	47
7.53	MPC	18	9.03	M2PC	48
7.53	MPC	19	9.03	M2PC	49
7.60	MPC	20	9.06	MPC	50
7.60	MPC	21	9.12	M2PC	51
7.98	MPC	22	9.20	M2PC	52
8.02	MPC	23	9.21	M2PC	53
8.05	MPC	24	9.23	M2PC	54
8.09	MPC	25	9.25	M2PC	55
8.26	M2PC	26	9.26	M2PC	56
8.29	MPC	27	9.33	M2PC	57
8.29	M2PC	28	9.34	M2PC	58
8.44	M2PC	29	9.39	M2PC	59
8.49	M2PC	30	9.63	M2PC	60

they are independent samples, this fact should be indicated using the argument “paired = FALSE”.

The resulting p-value is 1.60e-09, well below $\alpha = 0.05$, so the null hypothesis is rejected. There is enough statistical evidence to conclude that the MPC controller is better than the M2PC based on the THD$_\alpha$ at a 95% confidence level.

VI. CONCLUSION AND FUTURE WORK

When the assumption of a normal distribution is not met, gathering a large sample size in real-world scenarios is often impractical or impossible due to resource constraints, time, or data availability. In such cases, the Mann-Whitney U Test can provide a reliable alternative. This non-parametric test does not require the data to follow a normal distribution, making it suitable for a broader range of situations.

The Mann-Whitney U Test is especially powerful when at least one of the samples deviates from a normal distribution. It can effectively compare the performance between two controllers without the normality assumption. This test can assess any performance metric or characteristic if the samples are independent. Its versatility makes it a valuable tool for comparative evaluation in various scenarios.

Even when both samples are normally distributed, the Mann-Whitney test demonstrates power equivalent to the t-test for sample sizes of 20 or more, showcasing its efficacy as an alternative and a potentially preferable option in some cases.

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as an alternative and a potentially preferable option in some cases.

The Mann-Whitney U Test is limited to comparing only two populations. Future research could explore methods to extend the comparison to more than two groups. This could involve integrating parametric and non-parametric approaches to address a broader range of comparative studies.

The Mann-Whitney U Test goes beyond comparing current control strategies. It has broader implications for evaluating various control techniques like robotics, automotive control systems, power electronics, and industrial automation. This test is valuable for handling non-normally distributed data and providing meaningful comparisons under less restrictive conditions. It can be integrated with other non-parametric or parametric methods to create hybrid approaches for comprehensive analysis across more than two groups or conditions. This further expands its usefulness in emerging areas where traditional assumptions do not hold.

Mann-Whitney U Test is the preferred method for comparing the performance of two controllers, especially when traditional parametric assumptions are not met. It is robust, flexible, and powerful, making it suitable for various applications. However, its applicability is currently limited to two-sample comparisons.

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