# Comparative Study of Tire Models Applied to Electronic Stability Control in Automotive Simulator

Walter Paschoal, Igor Souza, Lucas Torres, André Murilo, and Renan Ozelo

***Abstract*—Automotive tires are crucial in vehicle dynamics, generating essential forces between the pavement and the vehicle. Active safety systems like Electronic Stability Control (ESC) rely on accurate tire force models. This paper presents a comparative analysis of the Pacejka Magic Formula (reference model), the brush model, and a proposed gain-saturation model using a single-track (bicycle) model with three degrees of freedom to evaluate lateral dynamics. Simulations conducted with a 14 Degree-of-Freedom (DOF) vehicle in VI-CarRealTime (VI-CRT) and analyzed in MATLAB revealed a significant correlation between simpler models and the benchmark reference for most relevant lateral vehicle dynamic variables, highlighting their capabilities and limitations through transient and stationary maneuvers. Simulation scenarios of the closed-loop ESC control system with the proposed tire models were carried out in real-time automotive software to compare performance with ESC homologation maneuver.**

**Link to graphical and video abstracts, and to code: https://latamt.ieeer9.org/index.php/transactions/article/view/8948**

***Index Terms*—Lateral Dynamics, Pacejka Magic Formula, Brush Model, Gain Saturation Model, Automotive Simulator, Electronic Stability Control**

## I. Introduction

Automotive tires play a significant role in the ride and dynamic behavior of the vehicle. Since the advent of radial tires in the 1970s, many functions are expected to be fulfilled by these rubber-made parts. Such functions include supporting vehicle load, absorbing road irregularities, and providing the vehicle-to-road interface. For the study of lateral dynamics, one function stands out: road surface friction. The traction and the ability to turn corners result from the force generated from the friction between the highway and the tires [1].

Systems like Electronic Stability Control (ESC), Electronic Stability Program (ESP), Dynamic Stability Control (DSC), or anti-lock braking system (ABS) rely on tire models to work properly. Active safety systems such as the Vehicle Dynamics Controller (VDC) act in standard and limit situations and require increasing accuracy in the description of dynamic reactions of tire contact forces [2], [3]. ESCs are proven active safety systems: authors in [4] estimated that when equipped with these systems, there is a reduction of up to 50 percent in deaths associated with car accidents. These safety systems actuate regarding the difficulty for a human driver to control vehicle lateral dynamics under extreme conditions. When the vehicle side-slip angle is large, the tire-force saturation causes difficulty in generating a yaw moment through the steering angle. Therefore, most of these systems work by constantly monitoring vehicle parameters (yaw rate, lateral acceleration, and longitudinal velocity) while comparing them to the desired trajectory based on the driver's inputs (steering angle, pedal position, or brake pressures). Then, if needed, the control action is applied through the manipulation of tire-traction forces to follow the desired motion under certain stability constraints.

To develop a reliable stability control system, variables such as yaw rate and side slip angle must be determined or measured. To calculate, estimate, or restrict such parameters, one of the system's inputs must be the tire forces. The description of the forces is done by tire models, which are correlated to two parameters, cornering stiffness, and the peak friction force/grip coefficient [5]. It describes a force/slip curve, necessary for the vehicle dynamics study and, therefore, for the active safety systems control operation. Models will be further explored in the next section of this paper.

With the advent of active safety systems, there is a growing interest in estimating the force generation by tires to optimize these systems. Authors in [6] propose an adaptive tire model based on the Magic Formula, considering the effects of changes in the tire's operating conditions for optimized control system use. In [7], the authors estimate the friction coefficient between a tire and the road using the brush model, demonstrating a good correlation between calculated and measured forces. The study in [8] focuses on estimating road friction for active safety systems by developing nonlinear state observers based on the brush tire model and validating the results using Carsim software. The paper in [9] also relies on the Magic Formula to create a tire-road friction coefficient estimation that meets control demands. In [10], a model predictive controller (MPC) integrates a double-track vehicle model and a combined-slip tire model to enhance vehicle stability. This controller optimizes tire slip ratios based

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on predicted lateral and longitudinal forces, ensuring stability under various conditions, with validation through simulations and road tests.

Research efforts to improve control strategies for correcting the handling stability of electric vehicles often utilize vehicle and tire models, particularly the Pacejka tire model, to compute necessary forces for control actuation. Authors in [11] present a coordinated control strategy that enhances the handling performance and stability of distributed-drive electric vehicles by integrating Torque Vector Control (TVC) and Electronic Stability Control (ESC). Similarly, the authors in [12] focus on improving the handling stability of distributed-drive electric vehicles using a seven-degree-of-freedom vehicle-dynamics model and the Magic Formula tire model. The study in [13] evaluates different tire models, including Fiala and the Magic Formula, determining the most suitable model for accurate vehicle performance evaluations, enhancing the understanding of how different models influence vehicle dynamics. Additionally, authors in [14] present an innovative approach to estimate tire forces essential for the Electronic Stability Control (ESC) system, validating their estimation methods through co-simulation using Matlab and CarSim software, as well as Hardware-in-the-Loop (HIL) testing. This indicates a growing interest and necessity in researching tire force generation for ESC applications.

Even though complex tire models can lead to good tire-road friction and vehicle side slip estimation, there is a drawback regarding computing power, as it might exceed the control cycle time. Authors in [15] affirm that complex models such as the Pacejka Magic Formula have trigonometric and exponential functions associated with several coefficients, which might slow down and exceed the simulation time. In such context, it is feasible to say that nonlinear models such as the Pacejka Magic Formula impose higher complexity that might lead to simulation timeout.

In this paper, a comparative study is proposed between three different tire models for analyzing relevant variables that considerably affect vehicle lateral dynamics. The vehicular mathematical model was developed considering 2 and 3 degrees of freedom (DOF) based on the single-track model (bicycle model). The study considers Pacejka's Magic Formula as a reference model applied in a real-time automotive simulation software VI-CarRealTime (VI-CRT), using tire setups that are consistent with real-world conditions. The main contribution of this paper consists of demonstrating the degree of correlation between simplified tire models and more complex representations, and how this can enable and enhance the implementation of control systems such as the ESC function. The aim is to show that automotive control systems can benefit from the similar dynamic behavior of simplified tire models without compromising their overall performance.

This paper is organized as follows. The second section presents the theoretical base of this work, which consists of the vehicular model used in this work (single track model) as well as an introduction to the tire models. The third section presents the experimental setup as well as the optimization routine on the simpler tire models. The fourth section shows the correlation of the tire models through the lateral dynamic single-track model and presents the results obtained. The last section is the conclusion, which presents a summary of the work developed while suggesting further development for future works.

## II. Vehicle Dynamics and Tire Models

The vehicular dynamic modeling can be divided into longitudinal dynamic, which represents the acceleration and braking, the vertical dynamic, which consists of road excitation frequency responses and comfort-related issues, and lateral dynamic which is related to steering and maneuverability of the vehicle [5]. This paper focuses on the lateral dynamic to validate the effectiveness of the single-track model and discuss a simpler approach to the tire forces modeling subject.

Seeking to evaluate the tire models, a precise representation of the car is essential to the work presented in this article, regarding the advantages and limitations of the three tire models analyzed.

The method chosen in this work is the single-track model because of its simplicity while maintaining reasonable fidelity. Considering small steering angles and, therefore, small tire slip angles, it is possible to use a single notation for the inner and outer tires. By doing so, the vehicle model is simplified to single front and rear tire angles. Single track models of two (lateral motion and yaw rate) and three degrees (lateral motion, yaw rate, and roll) of freedom were developed to apply the tire models (linear model with saturation, brush model, and Pacejka Magic formula).

The mathematical modeling of the single-track model proposed was based on the work of [5] and [16]. The single-track model shown below (Fig. 1) is adopted in this work.

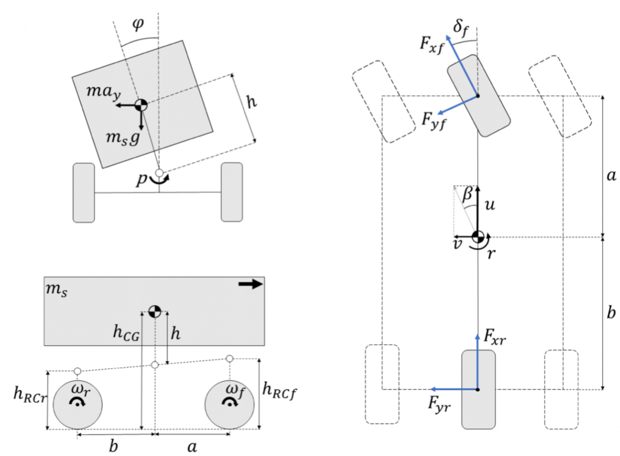


Fig. 1. Vehicular dynamic model adopted in this work. (Left) Lateral dynamics with roll angle, roll center height, and lateral acceleration. (Right) Bicycle model showing front steering angle, vehicle velocities, yaw rate, and tire forces.

Shown in the Fig. 1 is the distance from the center of gravity to the front axle ($a$) and to the rear axle ($b$), $h_{CG}$ the height of the center of gravity, $h_{RCf}$ and $h_{RCr}$ represent the height of roll axis (front and rear), $h$ is the distance of the roll axis to the center of gravity, $\delta_f$ steering angle, $u$ represents the longitudinal velocity and $v$ the lateral velocity. $\beta$ represents the side-slip angle, $r$ the yaw rate, $p$, the roll rate, $\phi$, the roll angle, $\omega_f$ and $\omega_r$ indicate the angular velocity of the front and rear wheels, $F_{xf}$ and $F_{xr}$ represents the longitudinal forces of front and rear tires and $F_{yf}$ e $F_{yr}$ represents the lateral forces of front and rear tires.

According to the references, Lagrange's equations are used to derive the equations of motion. Through the mathematical manipulation for the variables $y$, $v$, $r$ e $\phi$, which is not the focus of this work and it is described in the references adopted, we get to the final equation for three DOF:

$$\begin{cases} \Sigma F_y = mu_0(\dot{\beta}+r) + m_s h\dot{p} \\ \qquad = F_{yf}\cos\delta_f - bF_{yr} \\ \Sigma M_z = I_z\dot{r} + I_{ZZ}\dot{p} = aF_{yf}\cos\delta_f - bF_{yr} \\ \Sigma M_x = I_x\dot{p} + m_s h u_0(\dot{\beta}+r) + I_{ZZ}\dot{r} \\ \qquad = (m_s gh - K_\phi)\phi - C_\phi\dot{\phi} \end{cases} \tag{1}$$

Equally as important as defining the vehicle model is modeling the force generation of the tires, which is described by tire models. The choice of tire models is crucial to show a reliable force description, which is needed for many applications such as the electronic active safety systems (ex. ESC). It is wanted to describe the forces as close to reality as possible, to ensure the good function of such systems, in the most diverse situations. In this context, the following topics present different types of tire models that will be used to evaluate the bicycle model presented previously and applied in the stability control closed-loop.

### *A. Pacejka's Magic Formula*

In order to perform the analysis proposed by this paper, two models were chosen and a gain-saturation model was proposed. The first model chosen was the Magic Formula, which is, according to [17] an empirical tire model including five parameters and the most used of all models. This model shows a good correlation to the experimental data, often being treated as the benchmark. The mathematical model of the Magic Formula was based on [18]. The equation 3c describes the lateral tire force.

$$\alpha_y = \alpha + S_{hy} \tag{2a}$$
$$\phi_y = B_y\alpha_y - \arctan(B_y\alpha_y) \tag{2b}$$
$$F_y = D_y\sin(C_y\arctan(B_y\alpha_y - E_y\phi_y)) + S_{vy} \tag{2c}$$

The term $B_y$ represents the rigidity parameter, $C_y$ is the shape parameter, $D_y$ is the peak curve parameter and their mathematical product equals the rigidity at the zero lateral displacement point. The term $E_y$ in the Magic Formula tire model represents the curvature factor for lateral force. The last two parameters, $S_{hy}$ e $S_{vy}$. are vertical and horizontal fits, respectively, that indicate inherent residual forces of the tire's construction. These factors were not taken into consideration for the study proposed by this paper.

### *B. Brush Model*

Nonlinear models, such as the Pacejka Magic Formula, introduce higher complexity, which can lead to excessive computational demands in some simulations. To address this issue, we propose comparing and correlating this model with two others. The second model selected is the brush model, a physical representation that offers greater flexibility in parameter manipulation. It consists of a rigid ring connected to the road surface by bristles, which are perpendicular to the contact surface at the leading edge, as illustrated in Fig.2.

The brush model is designed to capture the nonlinear characteristics of tires by modeling the friction saturation phenomenon. The stiffness of the bristles reflects the actual stiffness of the tire's rubber elements, while the saturation phenomenon represents the frictional limits between the tire and the road surface (fig.2).

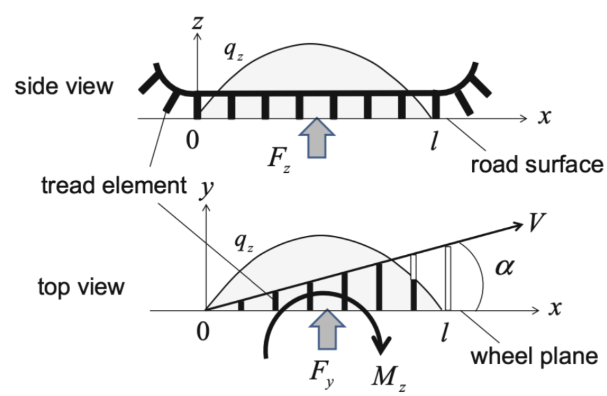


Fig. 2. Side and top view of the brushes while in contact with the ground. (Top) Tire tread elements with vertical force and deflection over contact length. (Bottom) Top view showing lateral force, aligning moment, and slip angle.

When the force at the tread element is less than the maximum frictional force, the tread element adheres to the contact surface. Otherwise, the tread element slips on the contact surface [19]. A parabolic force distribution is assumed at the contact patch of the tire and its equation is shown below.

$$q_z = \frac{F_z x}{bl^2}(1 - \frac{x}{l}) \tag{3}$$

Where $F_z$ represents the normal force, $b$ is the contact width, $l$ denotes the contact length and $x$ is the distance variable that is defined longitudinally in the contact patch. The lateral force described by this model is related by the equations below.

$$F_{adhesion} = b\int_0^{l_h} C_y x|\tan\alpha|dx \tag{4a}$$
$$F_{sliding} = \int_{l_h}^{l} \frac{6\mu F_z}{l^3}x(l-x)dx \tag{4b}$$

The term $l_h$ is the point that splits the adhesion and sliding regions, $\mu$ is the friction coefficient, $\alpha$ is the slip angle and $C_y$ is the bristle stiffness.

### *C. Gain-Saturation Model*

The last model proposed by this paper is a simple gain-saturation iteration. It consists of a model that remains linear up to a certain point, when the lateral force saturates to a

TABLE I
VEHICLE PARAMETERS FOR SIMULATION

| Parameter | Symbol | Value |
|---|---|---|
| Total mass | $m$ | 1986.6 kg |
| Suspended mass | $m_s$ | 1760.3 kg |
| Distance from CG to front axle | $a$ | 1.337 m |
| Distance from CG to rear axle | $b$ | 1.537 m |
| Moment of inertia (longitudinal) | $I_{xx}$ | 695.65 kg·m² |
| Moment of inertia (vertical) | $I_{zz}$ | 3564.73 kg·m² |
| Product of inertia | $I_{xz}$ | -0.734 kg·m² |
| Wheel inertia | $I_w$ | 1.389 kg·m² |

TABLE II
COEFFICIENTS FOR LATERAL FORCE CALCULATION AT $F_z = 4850\ N$

| Parameter | Symbol | Value |
|---|---|---|
| Stiffness factor | $B_y$ | 12.3732 |
| Shape factor | $C_y$ | 1.3507 |
| Peak factor | $D_y$ | 50872 |
| Curvature factor | $E_y$ | -0.0821 |
| Friction coefficient | $\mu_y$ | 1.0489 |
| Horizontal shift parameter | $S_{Hy}$ | 0 |
| Vertical shift parameter | $S_{Vy}$ | 0 |

limit, representing the maximum possible lateral force that can be generated by the tires. This model is structured as follows:

$$F_y = \begin{cases} C_\alpha \alpha & \text{if } F_y < \mu F_z \\ \mu F_z & \text{if } F_y \geq \mu F_z \end{cases} \quad (5)$$

This model assumes that, for small values of $\alpha$, the lateral force $F_y$ is proportional to the slip angle $\alpha$. This assumption underscores a crucial concept: the cornering stiffness, denoted as $C_\alpha$. The tire's cornering stiffness is one of its most critical parameters, significantly affecting the vehicle's handling and stability. [16]. By definition, it is the slope of the curve $F_y = F_y(\alpha)$ at $\alpha = 0$ for a given $F_z$.

The gain-saturation model offers simplicity, computational efficiency, and clear insights into the basic relationship between slip angle and lateral force. However, since it neglects non-linearities effects, it provides a simplified view of load dependency, which may diverge from more complex models in some analyses.

## III. METHODOLOGY

This section describes the vehicle and reference tire model used in the study. It includes the vehicle's specifications, such as dimensions, mass properties, and suspension characteristics, as well as the key parameters of the reference tire model, including vertical stiffness, damping coefficients, and Magic Formula tire model coefficients.

### A. Vehicle Model

The simulations in this article use the "Sedan" model from *VI-CarRealTime*, which has typical measurements for an average sedan. The vehicle's total mass, suspended mass, distances from the center of gravity to the front and rear axles, moments of inertia, and wheel inertia are summarized in Table I. These parameters are critical in vehicle dynamics analysis as they influence how the vehicle responds to steering, braking, and acceleration inputs.

### B. Reference Tire

In the Table II, the terms $B_y, C_y$, $D_y$, $E_y$, and $\mu_y$ represents constants calculated from the tire file coefficients using the equations taken from the reference [18], PAC2002 model and also the data from the software *VI-CarRealTime* for the reference sedan vehicle. The $S_{hk}$, $S_{vx}$ terms were modified to not influence the tire forces, once the single-track vehicle model does not consider suspension compensations for tire conicity and ply-steer.

It is worth noting that the tire model taken as the reference correlates with a physical tire, meaning that despite being a simulated tire, its characteristics were validated against a real tire by the authors. This ensures that the simulations performed with this tire model exhibit behavior very close to that of a physical tire, guaranteeing the accuracy and reliability of the results obtained.

### C. Tire Models Correlation

To coherently utilize the brush and gain-saturation models, optimization of certain parameters of the tire models was necessary. For the brush model, the parameters defining the contact patch were fixed, and the stiffness value of each bristle was optimized using the least squares estimation technique in relation to the reference curve obtained through the Pacejka model. For the gain-saturation model, the proportional gain value was optimized using the same reference curve, applying the weighted least squares estimation technique. In the region from 0º to 1º slip ratio, a weight of 20 was assigned, and beyond 1º lateral slip, a weight of 1 was assigned, also using the least squares estimation technique relative to the reference curve.

Mathematically, this can be expressed as finding the parameters $\theta$ that minimize the sum of the squared differences between the measured forces $F_{y,\text{measured}}$ and the predicted forces $F_{y,\text{predicted}}$:

$$\theta = \arg\min_\theta \sum_{i=1}^{n} (w \cdot F_{y,\text{measured},i} - w \cdot F_{y,\text{predicted},i})^2$$

Here, $\theta$ represents the reference variable (the force) used to optimize the bristle stiffness for the brush model and the cornering stiffness for the gain-saturation model, $n$ is the number of data points, and $w$ is the weighting function that is equal to 1 for all slip angle range for the brush model and for the gain-saturation model the $w$ function assumes the value of 20 from 0º to 1º slip angle and the value of 1 while beyond 1º lateral slip. The Coulombian friction theory is applied in brush and gain-saturation models aside from Pacejka model that is an experimental fitting with the tire coefficients.

The Fig. 3 shows the fit between the curves of the Pacejka model (blue line), the optimized brush model (orange line), and the optimized gain-saturation (yellow line). Authors in [20] also compared tire models (Dugoff, LuGre and Brush to

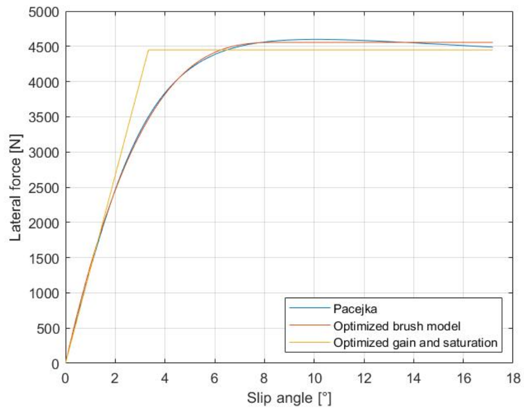


Fig. 3. Curves of different tire models optimized to Pacejka (reference model).

Pacejka Magic Formula) in a control system context, but for longitudinal dynamic. They found a good correlation between the models analyzed, especially in small slip values, having a minor offset for bigger slip quantities. In [21], authors compared Magic Formula to a linearized variation, also seeking to simplify the tire forces for specific control system operation. They also find a good approximation at small tire slip angles. In the next section, the analysis of the difference between the optimized models and the Pacejka will be conducted, through relevant variables to ESC systems.

## IV. Simulation Results and Lateral Dynamics Correlation with Tire Models

This section aims to identify the effects of different tire models on variables relevant to the control system. The general principle of a stability control system is to infer the desired vehicle motion from driver inputs, such as steering angle, and compare it to the actual motion measured by sensors. Due to the difficulty for a human driver to control vehicle lateral dynamics under extreme conditions, the control system intervenes to ensure the vehicle follows the desired trajectory. In this context, two variables play an important role: yaw rate and side slip angle. A high yaw rate indicates that the driver might be losing control of the vehicle, while a large side slip angle reduces the effectiveness of the steering angle in generating a yaw moment due to tire-force saturation [22].

### *A. Tire Models Correlation with Reference Tire*

The optimized tire models were applied to a three-degree-of-freedom (DOF) single-track model and compared to the reference Pacejka model. The 3 DOF model includes yaw, lateral motion, and the roll degree of freedom. Three maneuvers were used to analyze the motion: step steer, sweep steer, and sine with dwell, all with the influence of the ESC controller.

Fig. 4 illustrates the step steer maneuver, showing a satisfactory correlation between the optimized brush model, the optimized gain-saturation model, and the reference Pacejka model. There is little or no offset between the curves for both yaw rate and side slip angle (beta) in the first 1.5 seconds of motion. There is satisfactory tracking between the models even as the beta and yaw rate vary. After 1.5 seconds, there is a minor offset between the models. The optimized brush model showed the lowest side-slip angle, and the gain-saturation model had the lowest yaw rate after the curves stabilized.

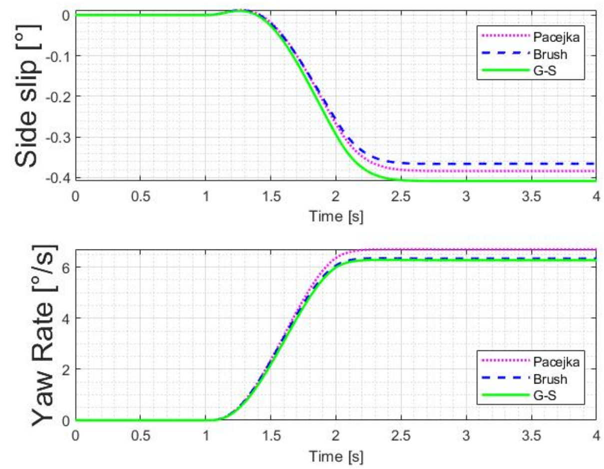


Fig. 4. Side slip (top figure) and yaw rate response (bottom figure) of different tire models to a step-steer input for 3 DOF vehicle model.

TABLE III
STEP STEER METRICS - SIDE SLIP

| Metric | Pacejka | Brush model | Gain-Saturation |
|---|---|---|---|
| Rise Time | 0.63s | 0.63s | 0.63s |
| Settling Time | 2.39s | 2.37s | 2.36s |
| Overshoot | 0.00% | 0.03% | 0.02% |

TABLE IV
STEP STEER METRICS - YAW RATE

| Metric | Pacejka | Brush model | Gain-Saturation |
|---|---|---|---|
| Rise Time | 0.64s | 0.64s | 0.64s |
| Settling Time | 2.08s | 2.06s | 2.07s |
| Overshoot | 0.05% | 0.21% | 0.18% |

The similarity between the tire models in the step steer metrics is shown in Tables III and IV.

Fig. 5 shows the yaw rate and side slip for the sweep steer maneuver. There is adequate correlation for both beta (side-slip angle) and yaw rate in this analysis. The curves for the proposed models (optimized brush model and optimized gain-saturation model) closely followed the reference (Pacejka model), with little to no offset.

### *B. Vehicle Model Comparison (VI-CRT)*

Given the good correlation between the tire models in the 3 DOF model, it was possible to plot the results for the same variables of interest using a 14 DOF automotive simulator model. The software VI Car Real Time provides output results from a complex vehicular model that is correlated to the actual vehicle. Fig. 6 shows the comparison for both 3 and 14 DOF models for side slip and yaw rate variables.

As seen in Fig. 6, there is similar tracking between the curves for the 3 DOF models compared to the 14 DOF

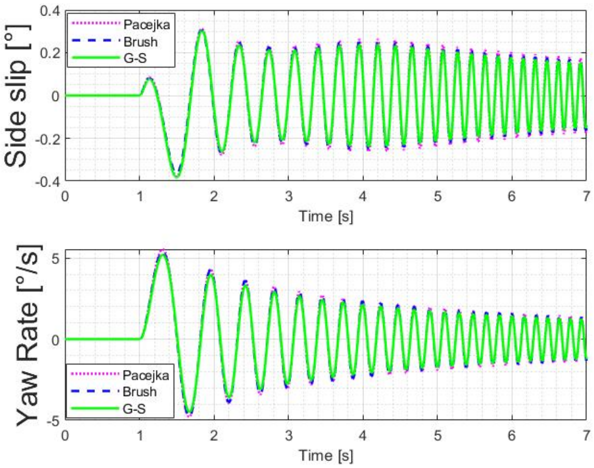


Fig. 5. Side slip (top figure) and yaw rate response (bottom figure) of different tire models to a sweep-steer input for 3 DOF vehicle models.

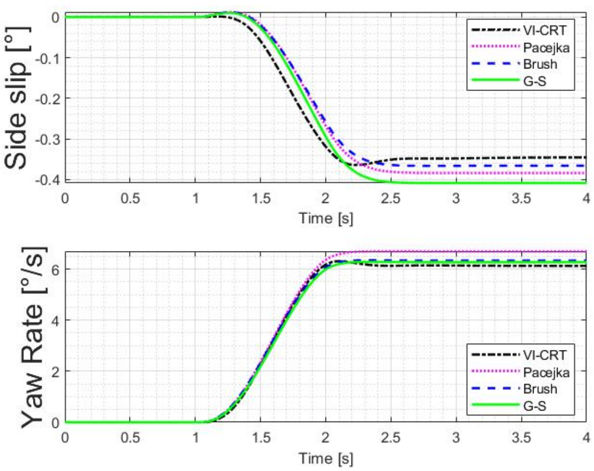


Fig. 6. Comparison of different tire models between 3 and 14 DOF models for yaw rate and side slip

TABLE V
STEP STEER CORRELATION STATISTICS

| $R^2$ | Pacejka | Brush model | Gain-Saturation |
|---|---|---|---|
| Side Slip | 82.06% | 86.14% | 71.34% |
| Yaw rate | 85.75% | 97.11% | 95.48% |

complex model. Despite an overshoot in the more complex model between the 2-2.5 second time step, both the initial first second and the period after 2.5 seconds show good curve correlation and tracking with little offset. These preliminary results indicate the potential use of simpler models, even in more realistic scenarios.

### *C. Simulation with the ESC Controller*

The Sine with Dwell maneuver involves precise steering inputs and measurements of yaw rate and lateral displacement to ensure that the ESC system provides adequate stability and control under simulated emergency conditions. This test is crucial for homologating ESC systems, ensuring they meet the required safety standards [23]. In order to evaluate the effectiveness of the proposed tire models to replace the reference in this scenario, four simulations were conducted: the reference Pacejka (ESC on and off), Brush model and GS model (both with the ESC ON).

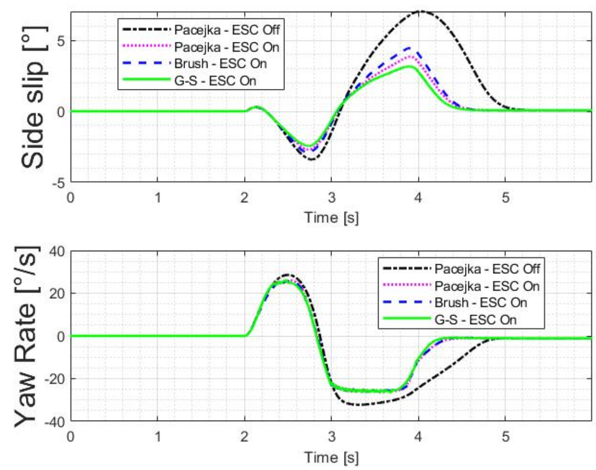


Fig. 7. Side slip (top figure) and yaw rate response (bottom figure) comparison with and without ESC activation during a sine with dwell input.

Fig. 7 shows the side slip and yaw rate for the sine with dwell maneuver. It is important to notice the difference in vehicle behavior for both studied variables between scenarios with and without ESC control (Pacejka - ESC off, used as a reference). Scenarios with ESC present a faster response and better control of the vehicle, representing more safety.

Analyzing the tire's influence, the three tire models produce considerable differences in the side slip variable during the second part of the maneuver. The brush model registers the highest peak, followed by the Pacejka model, and the gain-saturation model has the lowest peak. However, this phenomenon is not reproduced in the yaw rate variable, where all tire models exhibit almost the same behavior. These results suggest that the tire models primarily affect the side slip variable. Both non-linear models presented greater amplitudes, while the linear (gain-saturation) model produced the safer vehicle response

## V. CONCLUSIONS AND FUTURE WORKS

This study highlights the importance of tire dynamics in automotive control systems, using a single-track vehicle model to compare two tire force models: the physical brush model and the gain-saturation model. Both were optimized and compared to the more complex Pacejka Magic Formula to assess their accuracy in representing tire forces.

Preliminary results show that simpler models can effectively approximate the Pacejka model, especially in initial motion phases, offering benefits like faster computations and improved safety. Further research should validate these findings with more complex models and experimental data for broader application in control systems.

## ACKNOWLEDGMENTS

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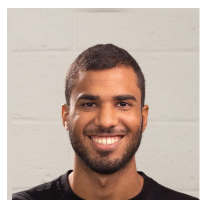

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