Simplified SEP Approximations of Coherent Digital Modulation Schemes over $\alpha - \kappa - \mu$ Fading Channel

Shreya Tated, Garv Anand, and Dharmendra Sadhwani

***Abstract*—In this paper, we propose novel, simplified yet tight approximations of the error probability expressions of numerous digital modulation schemes over a popular $\alpha - \kappa - \mu$ fading channel. With the help of a suitable approximation of the Gaussian Q function and utilization of Taylor's series expansion, we facilitate the cumbersome integrals which play a key role in simplification of the performance evaluation metrics like symbol error probability (SEP) of various digital modulation schemes. This facilitates cost effective receiver's design making the overall system economically viable. We further illustrate the accuracy of the proposed SEP expressions with the help of the relative error. An insight on the truncation error (and its upper bound) is also highlighted in this paper. We also compute the relative error in the upper bound of the truncation error to further justify the accuracy of the proposed integrals. Moreover, the asymptotic expressions for the integrals are also provided which gives an idea regarding diversity order of the wireless communication systems for large signal to noise ratios.**

Link to graphical and video abstracts, and to code: https://latamt.ieeer9.org/index.php/transactions/article/view/8684

***Index Terms*—α-κ-μ fading model, bit error rate, symbol error probability, modulation, Gaussian Q function, wireless communication, multipath fading, performance analysis and bounds**

I. Introduction

Wireless communication systems are obstructed by fading as well as shadowing. The latter is responsible for long term signal fluctuations whereas the former causes small scale fading. Hence, it is imperative to take the effect of fading while computing the performance evaluation metrics in communication theory. For example, on the basis of bit error rate (BER), the authors in [1] evaluated the performance of filtered multi-tone (FMT) modulation scheme under multipath fading. Furthermore, using the characteristic function (CHF) method, the authors in [2] computed the BER of various coherent as well as non-coherent digital modulation techniques over generalized fading model. To provide a deeper insight into the error performance analysis, the authors in [3] evaluated the BER of spacial modulation (SM) under rapidly time varying multipath fading. Noteworthy, there are various basic mathematical models which can represent multipath fading viz. Rayleigh, Nakagami$-q$, one-sided Gaussian, Nakagami$-m$, Rician [4]. Several attempts have been made to put all these basic fading models under one umbrella giving rise to generic fading distributions like $\kappa - \mu$, $\alpha - \mu$, $\eta - \mu$, $\lambda - \mu$, $\alpha - \mathcal{F}$ [5]–[8] which are extensively supported by the experimental results in practical communication systems.

Fraidenraich et al. in [9] introduced $\alpha-\eta-\mu$ and $\alpha-\kappa-\mu$ fading models which apart from including majority of the classical fading channels as stated above, prove to be significant in emerging communication systems. For example, the authors in [10], analyzed the performance of decode-and-forward multi-hop wireless communication systems in terms of the performance evaluation metrics like the amount of fading and channel capacity over non-linear generalized $\alpha - \kappa - \mu$ fading model. Kumar et al. in [11] analyzed the performance of cooperative spectrum sensing (CSS) over $\alpha-\eta-\mu$ and $\alpha-\kappa-\mu$ fading models. Further, in terms of error expressions, Kalia et al. in [12] analyzed the performance of spatial modulation systems over $\alpha - \kappa - \mu$ fading channel. Le et al. in [13] evaluated the performance of unmanned aeriel vehicle (UAV)-enabled wireless network over $\alpha-\kappa-\mu$ fading channel. Moualeu et al. in [14] tried to analyze the average error rate of digital modulation schemes over $\alpha-\kappa-\mu$ fading channel but they end up doing analysis for only those modulation schemes which involve the computation of first order of the Gaussian Q function [15, Eq. (1)] like binary phase shift keying (BPSK), binary amplitude shift keying (BASK) and binary frequency shift keying (BFSK). Moreover, it should also be noted that the analytical expressions derived in [14] have higher computational complexity due to the presence of the Fox H-functions. The author in [16] also analyzed the average error rate over $\alpha-\kappa-\mu$ fading model but for BPSK only. Moreover, the work of [16] is also expressed in terms of the Fox H-functions which again dilutes the analytical tractability of the error performance metrics. Salahat et al. in [17] also tried to analyze the error performance over $\alpha-\eta-\mu$ and $\alpha-\kappa-\mu$ fading models but the work is again limited to those modulation schemes where the computation of only first order of the Gaussian Q function is required and therefore the solution is not versatile which could cover all types of digital modulation schemes. A more general fading channel namely $\alpha-\eta-\kappa-\mu$ which includes $\alpha-\kappa-\mu$ and $\alpha-\eta-\mu$ as its special cases is explored by the authors in [18], [19] but in [18] the error performance metrics like average error rate is still expressed in terms of the complex Fox H-functions which again increases the computational complexity; while in [19], the average error rate is not expressed in closed form rather it requires numerical integration methods to compute a

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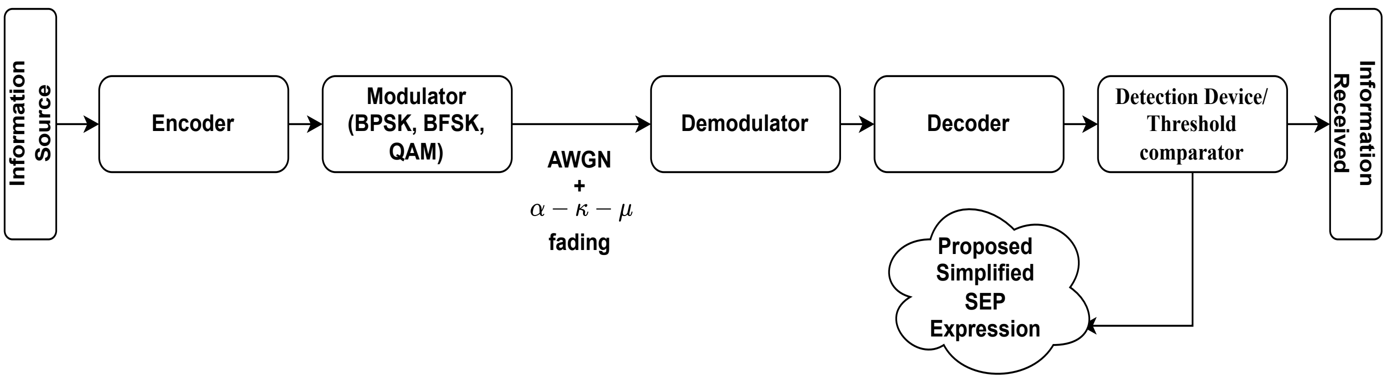

Fig. 1. Implementation of the Proposed Methodology in Wireless Communication System.

cumbersome integration.

As far as we know, via simplified analytical expressions, the detailed analysis of error performance of all the numerous digital modulation schemes known so far over $\alpha-\kappa-\mu$ fading model is still not reported in the literature. In this paper, we provide tractable and tight approximations to the symbol error probability (SEP) of all the well-known digital modulation techniques like BASK, BPSK, BFSK, square quadrature amplitude modulation (SQAM), BFSK with minimum correlation, rectangular-QAM (RQAM), hexagonal-QAM (HQAM) cross-QAM (XQAM) and differentially encoded quadrature phase shift keying (DE-QPSK) over $\alpha-\kappa-\mu$ fading distribution. Noteworthy, the derived analytical expressions are valid for the entire range of the fading parameters. Moreover, an insight on the closed form bounds of the truncation error is also highlighted in this paper which gives a fair idea regarding the accuracy of the proposed expressions. In addition, the asymptotic SEP is also included which highlights the diversity order (and error performance thereof) of communication systems over $\alpha-\kappa-\mu$ fading channel.

II. Simplified SEP Approximations of Coherent Digital Modulation Schemes

A. Channel Model

The probability density function (PDF) of $\alpha-\kappa-\mu$ fading channel is defined as [14]

$$f_\gamma(\gamma)=\sum_{j=0}^{\infty}\lambda_j\gamma^{\frac{\alpha}{2}(\mu+j)-1}e^{\frac{-\mu(1+\kappa)}{\bar{\gamma}^{\alpha/2}}\gamma^{\alpha/2}}, \tag{1}$$

where α,κ,μ are the fading parameters; $\lambda_j=\frac{\alpha\mu^{\mu+2j}\kappa^j(1+\kappa)^{\mu+j}}{2\Gamma(\mu+j)j!e^{\kappa\mu}\bar{\gamma}^{\frac{\alpha}{2}(\mu+j)}}$, $\bar{\gamma}$ is the average SNR, γ is the instantaneous SNR and $\Gamma(\cdot)$ is the Gamma function [20].

The $\alpha-\kappa-\mu$ fading model facilitates all the classical fading channels by just a simple variation of the fading parameters, as seen in Table I.

TABLE I
Classical Cases of the α-κ-μ Fading Channel [14]

Fading Channels	Parameters
κ-μ	$\alpha=2,\kappa=\kappa,\mu=\mu$
Rayleigh	$\alpha=2,\kappa\to 0,\mu=1$
Nakagami$-m$	$\alpha=2,\kappa\to 0,\mu=m$
One-Sided Gaussian	$\alpha=2,\kappa\to 0,\mu=1/2$
Rician	$\alpha=2,\kappa=K,\mu=1$

B. Simplified Yet Tight SEP Approximations

Proposition 1: We propose simplified approximated integrals which are vital in computing the SEP of numerous digital modulation schemes over $\alpha-\kappa-\mu$ fading channel, as

$$I_1\approx\frac{A}{8^p}\sum_{k_1+k_2+k_3+k_4=p}\sum_{j=0}^{L-1}\sum_{n=0}^{N-1}(-1)^n\lambda_j\frac{p!\mu^n(1+\kappa)^n}{n!k_1!k_2!k_3!k_4!\bar{\gamma}^{\frac{n\alpha}{2}}}\times\frac{\Gamma(\beta)}{\nu^\beta},\quad \text{for}\ \ \beta>0,\nu>0 \tag{2a}$$

$$I_2\approx\frac{B}{64}\sum_{l=1}^{16}\sum_{j=0}^{L-1}\sum_{n=0}^{N-1}\frac{(-1)^n\lambda_j\mu^n(1+\kappa)^n\Gamma(\beta)}{0.5^\beta n!\bar{\gamma}^{\frac{n\alpha}{2}}\delta_l^\beta},\quad \text{for}\ \beta>0,\delta>0 \tag{2b}$$

where $\beta=\frac{\alpha}{2}(\mu+j)+n\frac{\alpha}{2}$, $\nu=\frac{\sigma^2\sum_{i=1}^{4}\xi_ik_i}{2}$, ξ_i is defined in Table II; A, B, σ and ζ depend upon the type of the modulation scheme and the parameter δ_l is defined as: $\delta_l=[\xi_1(\sigma^2+\zeta^2),\xi_2(\sigma^2+\zeta^2),\xi_3(\sigma^2+\zeta^2),\xi_4(\sigma^2+\zeta^2),\xi_1\sigma^2+\xi_2\zeta^2,\xi_1\sigma^2+\xi_3\zeta^2,\xi_1\sigma^2+\xi_4\zeta^2,\xi_2\sigma^2+\xi_1\zeta^2,\xi_2\sigma^2+\xi_3\zeta^2,\xi_2\sigma^2+\xi_4\zeta^2,\xi_3\sigma^2+\xi_1\zeta^2,\xi_3\sigma^2+\xi_2\zeta^2,\xi_3\sigma^2+\xi_4\zeta^2,\xi_4\sigma^2+\xi_1\zeta^2,\xi_4\sigma^2+\xi_2\zeta^2,\xi_4\sigma^2+\xi_3\zeta^2]$.

It should be noted that since all the parameters α,μ,j,n,ξ,k are positive, the conditions $\beta,\nu,\delta>0$ always hold true yielding a versatile solution (2).

Proof: The integrals which are vital in the SEP computation of digital modulation schemes over $\alpha-\kappa-\mu$ fading channel are defined as [4]:

$$I_1=A\int_0^\infty Q^p(\sigma\sqrt{\gamma})\times f_\gamma(\gamma)d\gamma \tag{3a}$$

and

$$I_2=B\int_0^\infty Q(\sigma\sqrt{\gamma})Q(\zeta\sqrt{\gamma})\times f_\gamma(\gamma)d\gamma, \tag{3b}$$

TABLE II
PARAMETER RELATED TO GAUSSIAN Q FUNCTIONS'S APPROXIMATION [21]

i	ξ_i
1	2.627414291213963e+1
2	3.239828844061399e+0
3	1.446462700182916e+0
4	1.039566130751374e+0

where p is the integer order of the Gaussian Q function, $Q(\cdot)$. Clearly, Eq. (3a) is cumbersome to compute. Hence, we utilize a simple yet accurate alternative form for the Gaussian Q function with four sub-intervals, defined as [21]

$$Q^p(\sigma\sqrt{\gamma}) \approx \frac{1}{8^p}\left\{\sum_{l=1}^{4}\exp\left(-\frac{\xi_l\sigma^2\gamma}{2}\right)\right\}^p. \quad (4)$$

Using multinomial theorem, we simplify Eq.(4) as

$$Q^p(\sigma\sqrt{\gamma}) \approx \frac{1}{8^p}\sum_{k_1+k_2+k_3+k_4=p}\frac{p!}{k_1!k_2!k_3!k_4!} \times \exp\left(-\frac{\sigma^2\gamma\sum_{l=1}^{4}\xi_l k_l}{2}\right). \quad (5)$$

It should be noted that substituting Eqs.(1) and (5) into Eq. (3a) will lead to a cumbersome integral. Hence, to facilitate Eq. (3a), we use Taylor's series expansion of the exponential term present in Eq. (1), yielding:

$$\exp\left(\frac{-\mu(1+\kappa)}{\bar{\gamma}^{\alpha/2}}\gamma^{\alpha/2}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{\mu(1+\kappa)}{\bar{\gamma}^{\alpha/2}}\gamma^{\alpha/2}\right)^n. \quad (6)$$

Now using Eq. (6) and substituting Eqs. (5) and (1) into Eq. (3a), we have

$$I_1 \approx \frac{A}{8^p}\sum_{k_1+k_2+k_3+k_4=p}\sum_{j=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n\lambda_j\frac{p!\mu^n(1+\kappa)^n}{n!k_1!k_2!k_3!k_4!\bar{\gamma}^{\frac{n\alpha}{2}}} \times \int_0^{\infty}\exp\left(-\frac{\sigma_2^2\gamma\sum_{l=1}^{4}\xi_l k_l}{2}\right)\gamma^{\frac{n\alpha}{2}+\frac{\alpha}{2}(\mu+j)-1}d\gamma \quad (7)$$

Eq. (7) can be easily solved using the identity [20, (3.381/4)]:

$$\int_0^{\infty}\gamma^{\beta-1}e^{-\nu\gamma}d\gamma = \frac{\Gamma(\beta)}{\nu^{\beta}}, \quad \beta>0, \nu>0, \quad (8)$$

which after truncation to L and N terms yields the proposed approximated SEP integral Eq. (2a). We can similarly solve Eq. (3b) yielding Eq. (2b). This completes the proof. Hence, using Eq. (2), we can compute the SEP of numerous digital modulation schemes as exhaustively presented in Table III.

As far as we know, Eq. (2) is new and involves simple algebraic functions unlike [14] where the SEP is expressed as an infinite series of the Fox H-functions whose computational complexity is relatively high. On the other hand, besides being tractable, Eq. (2) easily converges to finite number of terms N and also requires fewer number of terms L to accurately compute the SEP. Apart from this, we have proposed the SEP for all the well-known coherent digital modulation schemes unlike [14] where the SEP of only BASK, BPSK and BFSK is proposed; whereas the proposed work includes the computation of well-known QAM schemes as well. This proves the versatility of the proposed solution.

C. Bounds for the Truncation Error of (2)

We get the proposed SEP integrals of Eq. (2) after truncated to finite N and L i.e. they must converge to ensure acceptable truncation. To do so, we provide the bounds for these integrals. We explicitly derive the truncation error for Eq. (2a) whereas the same can be similarly computed for Eq. (2b).

The truncation of Eq. (2a) after $N-1$ terms results in the following truncation error:

$$\in_{I_1} \approx \frac{A}{8^p}\sum_{k_1+k_2+k_3+k_4=p}\sum_{j=0}^{\infty}\sum_{n=N}^{\infty}(-1)^n\lambda_j \times \frac{p!\mu^n(1+\kappa)^n}{n!k_1!k_2!k_3!k_4!\bar{\gamma}^{\frac{n\alpha}{2}}}\frac{\Gamma(\beta)}{\nu^{\beta}}. \quad (9)$$

Now on changing the summation index to $t = n - N$ and using the identities $(t+N)! = N!(N+1)_t$, Eq. (9) can be written as:

$$\in_{I_1} \approx \frac{A}{8^p}\frac{(-1)^N\alpha(\mu(1+\kappa))^{\mu+N}e^{-\kappa\mu}}{2N!\bar{\gamma}^{\frac{\alpha(\mu+N)}{2}}}\sum_{k_1+k_2+k_3+k_4=p}\frac{p!}{k_1!k_2!k_3!k_4!} \times \sum_{j=0}^{\infty}\frac{\mu^{2j}\kappa^j(1+\kappa)^j\Gamma\left(\frac{\alpha(\mu+j+N)}{2}\right)}{\Gamma(\mu+j)j!\nu^{\frac{\alpha(\mu+j+N)}{2}}}\sum_{t=0}^{\infty}\frac{\left(\frac{\alpha}{2}(\mu+j+N)\right)_{\frac{\alpha t}{2}}}{(N+1)_t} \times \left(\frac{-\mu(1+\kappa)}{(\bar{\gamma}\nu)^{\frac{\alpha}{2}}}\right)^t, \quad (10)$$

where $(x)_y = \dfrac{\Gamma(x+y)}{\Gamma(x)}$ is the Pochhammer symbol [20, Page No. xliii]. Now, using the above definition of the Pochhammer symbol, the identity $(1)_t = t!$ and the definition ${}_2F_1(a_1,a_2;b_1;z) = \sum_{t=0}^{\infty}\frac{(a_1)_t(a_2)_t}{(b_1)_t}\frac{z^t}{t!}$ [20, Eq. (9.14.1)], Eq. (10) can be written as:

$$\in_{I_1} \approx \frac{A}{8^p}\frac{(-1)^N\alpha\left(\mu(1+\kappa)\right)^{\mu+N}e^{-\kappa\mu}}{2N!\bar{\gamma}^{\frac{\alpha(\mu+N)}{2}}}\sum_{k_1+k_2+k_3+k_4=p} \times \frac{p!}{k_1!k_2!k_3!k_4!}\sum_{j=0}^{\infty}\frac{\mu^{2j}\kappa^j(1+\kappa)^j\Gamma\left(\frac{\alpha(\mu+j+N)}{2}\right)}{\Gamma(\mu+j)j!\nu^{\frac{\alpha(\mu+j+N)}{2}}} \times {}_2F_1\left(1, \frac{\alpha(\mu+j+N)}{2}; N+1; \left(\frac{-\mu(1+\kappa)}{(\bar{\gamma}\nu)^{\frac{\alpha}{2}}}\right)\right), \quad (11)$$

where ${}_2F_1(\cdot)$ is the generalized hypergeometric function [20, Eq. (9.14.1)].

Finally, the summation with index j when truncated to $L-1$ terms gives the upper bound in the truncation error as:

TABLE III
PROPOSED SEP OF VARIOUS COHERENT DIGITAL MODULATION SCHEMES OVER α-κ-μ FADING CHANNEL

Coherent Modulation Schemes[1]	**Proposed Simplified SEP** (P_s)
BPSK [4]	$P_s \approx I_1\|_{p=1}$
BFSK [4]	$P_s \approx I_1\|_{p=1}$
BFSK with minimum correlation [4]	$P_s \approx I_1\|_{p=1}$
M-SQAM [4]	$P_s \approx I_1\|_{p=1} - I_2\|_{\sigma=\zeta}$
$M \times N$-RQAM [4]	$P_s \approx I_1\|_{p=1} - I_2\|_{\sigma=\zeta}$
$M \times N$-XQAM [22]	$P_s \approx I_1\|_{p=1} - I_2\|_{\sigma=\zeta}$
M-HQAM [23]	$P_s \approx I_1\|_{p=1} + I_2\|_{\sigma=\zeta=\sqrt{\frac{2\sigma}{3}}} - 3I_2\|_{\zeta=\sqrt{\frac{\sigma}{3}}}$
DE-QPSK [24]	$P_s \approx 4(I_1\|_{p=1} - 2I_1\|_{p=2} + 2I_1\|_{p=3} - 2I_1\|_{p=4})$

[1] SQAM: $A = 4(1-\frac{1}{\sqrt{M}}), B = 4(1-\frac{1}{\sqrt{M}})^2, \sigma = \sqrt{\frac{3}{M-1}}$; RQAM: $A = (4-\frac{2}{M}-\frac{2}{N}), B = 4(1-\frac{1}{M})(1-\frac{1}{N}), \sigma = \sqrt{\frac{12}{5M\times N-4}}$; XQAM: $A = (4-\frac{2}{M}-\frac{2}{N}), B = (4-\frac{4}{M}-\frac{4}{N}-\frac{8}{M\times N}), \sigma = \sqrt{\frac{96}{31M\times N-32}}$; HQAM: $A = 2(3-4M^{\frac{-1}{2}}+M^{-1}), B = 4(1-M^{\frac{-1}{2}})^2$, $\sigma = \sqrt{\frac{24}{7M-4}}$; DE-QPSK: $A=1, \sigma=1$; BPSK: $A=1, \sigma=\sqrt{2}$; BFSK: $A=1, \sigma=1$; BFSK with minimum correlation: $A=1, \sigma=\sqrt{1.43}$

$$\in_{I_1} < \frac{A}{8^p}\frac{(-1)^N\alpha\left(\mu(1+\kappa)\right)^{\mu+N}e^{-\kappa\mu}}{2N!\bar{\gamma}^{\frac{\alpha(\mu+N)}{2}}}\sum_{k_1+k_2+k_3+k_4=p} \times \frac{p!}{k_1!k_2!k_3!k_4!}\frac{\mu^{2L}\kappa^L(1+\kappa)^L\Gamma\left(\frac{\alpha(\mu+L+N)}{2}\right)}{\Gamma(\mu+L)L!\nu^{\frac{\alpha(\mu+L+N)}{2}}} \times {}_2F_1\left(1, \frac{\alpha(\mu+L+N)}{2}; N+1; \left(\frac{-\mu(1+\kappa)}{(\bar{\gamma}\nu)^{\frac{\alpha}{2}}}\right)\right). \quad (12)$$

Similarly, the closed-form upper bound for the truncation error of Eq. (2b) can be given as:

$$\in_{I_2} < \frac{B}{64}\frac{(-1)^N\alpha(\mu(1+\kappa))^{\mu+N}e^{-\kappa\mu}}{2N!\bar{\gamma}^{\frac{\alpha(\mu+N)}{2}}}\sum_{l=1}^{16}\frac{\mu^{2L}\kappa^L(1+\kappa)^L\Gamma\left(\frac{\alpha(\mu+L+N)}{2}\right)}{\Gamma(\mu+L)L!(2\delta_l)^{\frac{\alpha(\mu+L+N)}{2}}} \times {}_2F_1\left(1, \frac{\alpha(\mu+L+N)}{2}; N+1; \left(\frac{-\mu(1+\kappa)}{(2\bar{\gamma}\delta_l)^{\frac{\alpha}{2}}}\right)\right). \quad (13)$$

D. Asymptotic SEP

In order to get further insights on the systems' performance, asymptotic SEP is computed i.e. when $\bar{\gamma} \to \infty$. It can be easily verified that in the high SNR regime, $j = 0$ term of (1) dominates. Apart from this, Eq. (6) is simplified to:

$$\exp\left(\frac{-\mu(1+\kappa)}{\bar{\gamma}^{\alpha/2}}\gamma^{\alpha/2}\right) \approx 1. \quad (14)$$

Now carrying out the similar process as in subsection $'B'$, the asymptotic integrals can be written as:

$$I_1^{Asymp} \approx \frac{A}{8^p}\sum_{k_1+k_2+k_3+k_4=p}\lambda_0\frac{p!}{k_1!k_2!k_3!k_4!} \times \frac{\Gamma(\beta)}{\nu^\beta}, \quad (15a)$$

$$I_2^{Asymp} \approx \frac{B}{64}\sum_{l=1}^{16}\frac{\lambda_0\Gamma(\beta)}{2^\beta\delta^\beta}, \quad (15b)$$

where $\lambda_0 = \frac{\alpha\mu^\mu(1+\kappa)^\mu}{2\Gamma(\mu)e^{\kappa\mu}\bar{\gamma}^{\frac{\alpha\mu}{2}}}$ and $\beta = \frac{\alpha\mu}{2}$.

Now, with the help of Eq. (15) and Table III, the asymptotic SEP of various digital modulation techniques can be easily computed. Noteworthy, since I_1^{Asymp} and I_2^{Asymp} are proportional to $\bar{\gamma}^{-\frac{\alpha\mu}{2}}$, the diversity gain only depends upon α and μ.

III. SIMULATION RESULTS AND DISCUSSIONS

A. Significance of N and L in determining the accuracy of (2)

The accuracy of (2a) can be seen via Figs. 2-4 where we have computed the relative error (RE) in (2a) as:

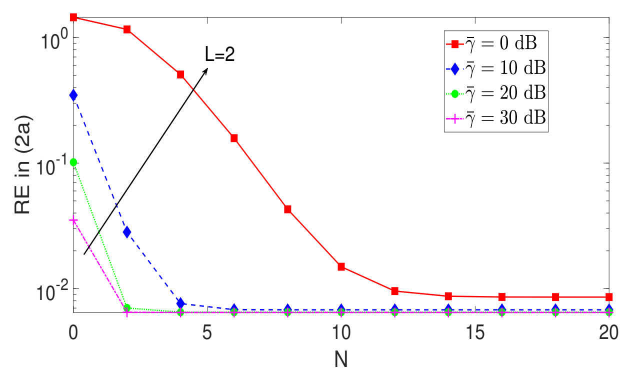

Fig. 2. Accuracy of Eq. (2a) for BPSK against N at various values of average SNRs with $\alpha = 1, \kappa = 1, \mu = 1$.

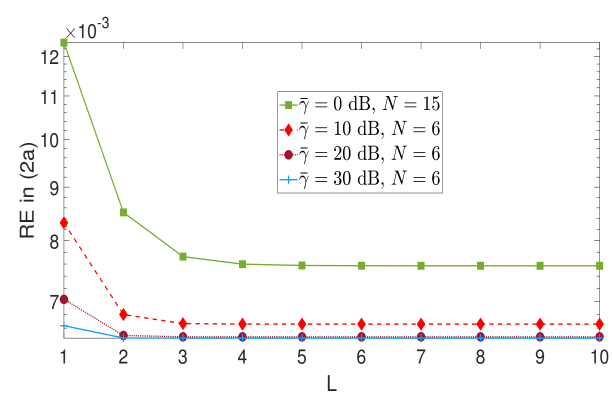

Fig. 3. Accuracy analysis of Eq. (2a) for BPSK against different values of L with $\alpha = 1, \kappa = 1, \mu = 1$.

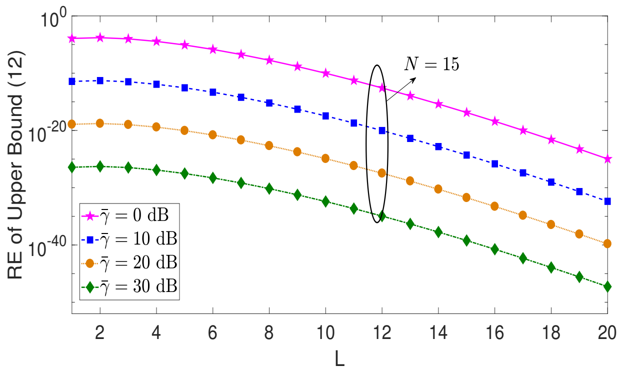

Fig. 4. Accuracy of Eq. (2a) for BPSK via computing the RE in the upper bound of the truncation error against different L with $\alpha = 1, \kappa = 1, \mu = 1$.

$$RE = \frac{|\text{Proposed Eq. (2a)} - \text{Exact Eq. (3a)}|}{\text{Exact Eq. (3a)}}, \quad (16)$$

whereas the RE in the upper bound of the truncation error of Eq. (12) can be calculated as $\frac{\in_{I_1} \text{Eq.}(12)}{\text{Exact Eq. (3a)}}$. As an instance, we have taken BPSK modulation for one set of the fading parameters: $\alpha = 1, \kappa = 1, \mu = 1$.

Fig. 2 illustrates that for very noisy environment ($\bar{\gamma} = 0$ dB), the RE saturates to 10^{-2} for $N \geq 15$. On the other hand, when $\bar{\gamma}$ increases, the same level of accuracy is achieved for lesser number of terms i.e. for $N = 2$. However, it should be noted that increasing the number of terms N does not increase the computational complexity as the proposed expressions are expressed in terms of simple algebraic functions which can be computed at hand. It should be noted that only $L = 2$ is sufficient for the RE to reach to as low as 10^{-2} i.e. 1%.

In Fig. 3, the RE is calculated for different values of L and it can be seen that after $L = 2$, the RE becomes as low as 10^{-3}. It should be noted that in case of a very noisy environment, $N = 15$ is needed to make a constant, low value of the RE; whereas the scenario where the effect of noise is less, $N = 6$ is sufficient to achieve the same level of accuracy.

In Fig. 4, the RE in the upper bound of the truncation error of Eq. (12) is computed, against various values of L. It can be seen that for $L = 2$ we get the RE as low as 10^{-5} (for $\bar{\gamma} = 0$ dB) which further decreases as $\bar{\gamma}$ and L increases.

It should be noted that a similar, thorough analysis of the RE was carried out for various digital modulation schemes against various combinations of the fading parameters. It was concluded that for $L = 9$ and $N = 145$ we get the desired accuracy for the SEP of all the digital modulation schemes as illustrated in Table III. Although it seems that a large value of N increases the computational complexity but since the proposed approximation involves only elementary functions, the computational time in the available software packages is negligible; which further gives an idea regarding the tractability of the proposed solution.

B. SEP Plots

In this section, the accuracy of Eq. (2) is shown by plotting the SEP vs average SNR using MATLAB software.

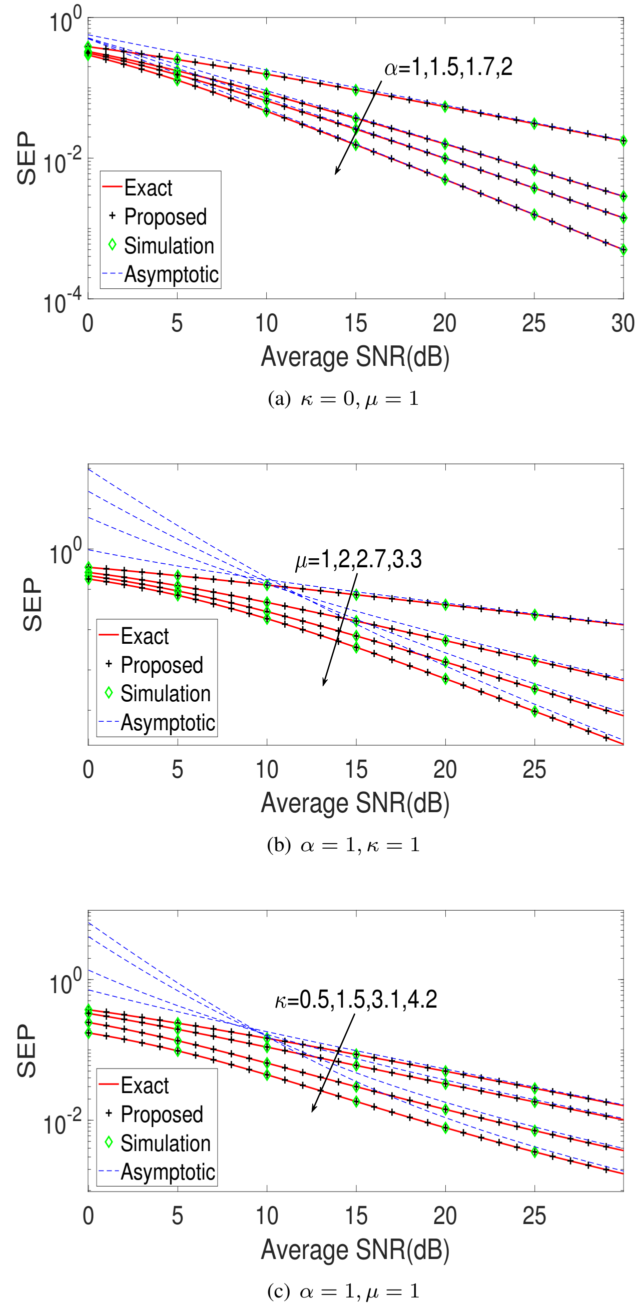

Fig. 5. Accuracy of Eq. (2a) for BPSK against numerous values of α, κ and μ.

To assess the impact of the fading on the effectiveness of the suggested system and channel model, in Fig. 5, we have included the SEP of BPSK for various values of α, κ and μ. It can be clearly seen that an increase in the value of one parameter keeping two parameters constant improves the system's error performance. It is noticeable that the approximated SEP curves are in remarkable agreement with the exact SEP plots for the entire range of the average SNRs, and for various values of the fading parameters. The accuracy of the proposed SEP is further verified with the help of simulated results. The asymptotic SEP also shows that as the value of any one fading parameter (say α) increases, the slope of

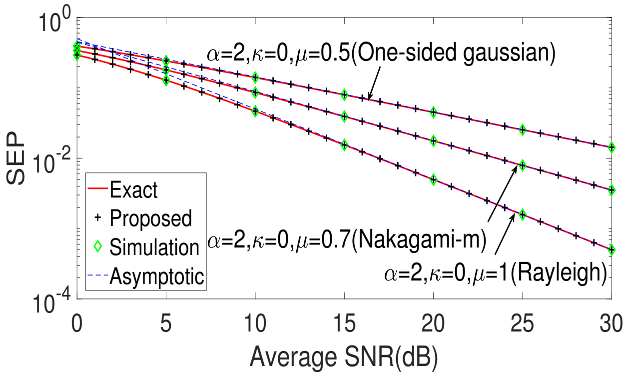

Fig. 6. Significance of Eq. (2a) in computing the SEP of BPSK for numerous classical channels of α-κ-μ fading distribution.

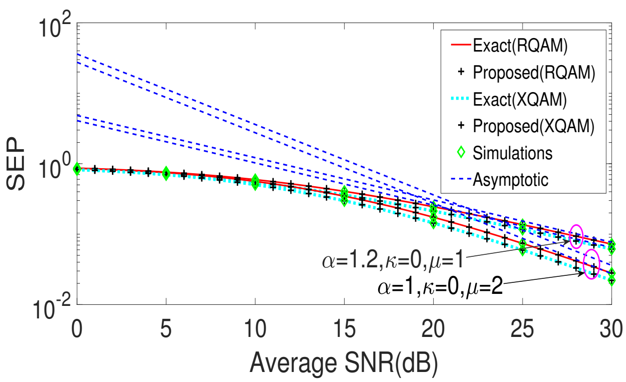

Fig. 7. SEP of 32-RQAM and 32-XQAM computed via Eq. (2) for different combinations of α, κ, μ.

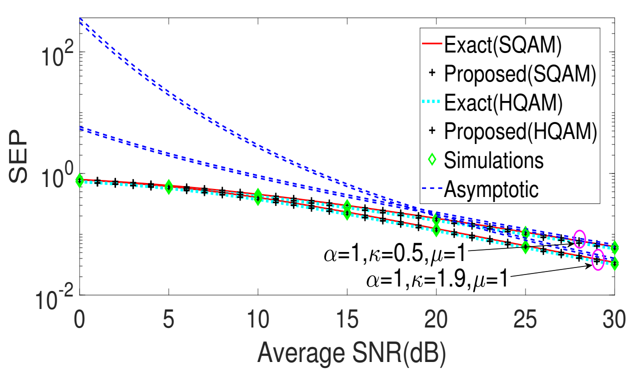

Fig. 8. Utility of Eq. (2) in computing the SEP of 16-SQAM and 16-HQAM for different combinations of α, κ, μ.

the proposed SEP becomes steeper which clearly indicates performance improvement of BPSK.

Fig. 6 further shows the SEP plots of BPSK for some of the classical distributions which are derived from α-κ-μ fading model, which in turn shows the versatility of the proposed solution. Here also, we can see that as the value of μ increases keeping $\alpha = 2, \kappa = 0$ constant, the error performance of BPSK is enhanced. Clearly, the exact, proposed and simulated curves are in excellent agreement highlighting the accuracy of the proposed analysis. Noteworthy, the asymptotic SEP indicates the performance improvement (with the help of steep slope) as the value of μ increases.

In Fig. 7, the usefulness of the proposed solution is further shown by computing the SEP of 32-RQAM and 32-XQAM against different combinations of α and μ keeping the value of $\kappa = 0$ constant. We can see that as the values of both α and μ increases, the error performance improves. It can also be seen that the results obtained from the analytical expressions are very tight for the entire average SNR range, and for all the considered values of the fading parameters. We can further verify that for both the combinations of the fading parameters, 32-XQAM yields power gain of around 1.139 dB over 32-RQAM [25]. The asymptotic SEP also matches with the proposed SEP curves for higher average SNR regime.

Similarly, via Fig. 8, the SEP plots of 16-HQAM and 16-SQAM are shown which closely match with the exact curves. We have shown the results for fixed values of $\alpha = \mu = 1$ and variable κ. Clearly, as we increase the value of κ, we get the reduced SEP (if we see at constant average SNR), indicating performance improvement. Further, from the proposed SEP curves, it can also be seen that 16-HQAM provides a power gain of around 0.45 dB [23] over 16-SQAM.

Noteworthy, all the analytical findings as shown in Figs. 5-8 have been verified with the help of Monte-Carlo simulations. For better understanding, we hereby explain the channel implementation through Monte-Carlo simulations. The simulation process can be broken down into the following steps:

- **Initialization**: The number of symbols to be simulated is set to 10,000,000. This large number is chosen to ensure that the results are statistically reliable. The SEP for different SNR values is initialized to zero.
- **SNR Loop**: For each SNR value (in dB), the following steps are performed:
  - **Channel Modeling**: The $\alpha-\kappa-\mu$ fading model is simulated. The channel coefficient is generated according to the $\alpha-\kappa-\mu$ model. The channel coefficient represents the effect of the channel on the transmitted signal, including path loss, fading, and phase shift. The channel coefficient is normalized to have a mean power of 1. This ensures that the channel does not introduce any additional gain or loss to the signal power.
  - **Signal Generation**: A random sequence of symbols is generated. These symbols are mapped to constellation points (as per the case i.e. BFSK, BPSK, and different QAM schemes). The constellation points represent the possible values that the transmitted signal can take.
  - **Noise Generation**: Noise is added to the signal. The noise power is calculated based on the average power of the signal and the current SNR value. The noise is complex Gaussian with zero mean and a variance equal to half the noise power (since it's complex noise). This represents the effect of thermal noise in the receiver.
  - **Signal Reception**: The received signal is the sum of the faded signal (channel coefficient times the signal) and the noise. This represents the signal that

the receiver observes.
  - **Detection**: For each received symbol, the closest constellation point is found, and the corresponding symbol is detected. This is done by minimizing the Euclidean distance between the received signal and the possible transmitted signals. This represents the receiver's attempt to estimate the transmitted symbol from the received signal.
  - **Error Calculation**: The SEP is calculated as the ratio of the number of incorrectly detected symbols to the total number of symbols. This gives a measure of the system's performance.
- **Results**: The SEP for different SNR values is returned. This provides an estimate of the system's performance over $\alpha - \kappa - \mu$ fading channels at different average SNR values. The randomness in the simulation comes from the random generation of the channel coefficients and the noise. By averaging the results over a large number of symbols, the law of large numbers ensures that the simulation results are close to the true theoretical performance.

IV. Conclusion and Future Work

Via this paper, a thorough analytical framework has been described for the simplified SEP approximations of numerous coherent digital modulation techniques over α-κ-μ fading channel. With the help of graphical illustrations, we have shown that the proposed analytical expressions converge for a finite number of simple algebraic terms giving results with minimum computational complexity. Besides being tractable, the derived expressions are tight for the entire range of the average SNR. In addition, the proposed solution yields results for the entire range of the fading parameters which are further validated using computer simulations.

As a future work, the approach using Taylor series expansion can be used to simplify the error performance metrics of several wireless communication systems over intractable yet significant fading distributions like multi-cluster fluctuating two-ray fading model [26] which includes several important fading distributions like Nakagami-m, fluctuating two-ray fading model, Rician shadowed, $\kappa - \mu$ shadowed, two-wave, as its special cases.

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