Analysis and Virtual Validation of Vehicle Dynamics Models for Electronic Stability Control

Lucas A. Torres, André Murilo, Renato V. Lopes, Vinicius Leal

***Abstract*—This work investigates the utility and efficiency of simplified vehicle dynamics models in the Electronic Stability Control (ESC) context. The focus is not merely on the correlation between simplified and complex models but on the added value such models can bring to the automotive industry. By comparing the simplified models based on bicycle representation with a high-fidelity VI-CarRealTime (VI-CRT) model, this paper demonstrates that the simplified models have good correlation and offer computational advantages. This approach addresses the key dynamics essential for ESC and considers a generic sedan vehicle model to simulate standard maneuvers widely performed by the automotive industry. All the proposed models are nonlinear and include tire modeling based on the Magic Formula, with parameters derived from experimental tests. The paper also introduces a methodology for parameter optimization, enhancing the model's accuracy and reliability. The results indicate that the simplified models can be a viable alternative for specific applications such as control strategy tuning, rapid prototyping, and early-stage development. This work aims to fill the gap in the literature concerning the practical applicability and limitations of simplified vehicle dynamics models, thereby making a significant contribution to the field.**

Link to graphical and video abstracts, and to code: https://latamt.ieeer9.org/index.php/transactions/article/view/8394

***Index Terms*—Vehicle dynamics, Electronic Stability Control, Vehicle Model, VI-CarRealTime.**

I. Introduction

Vehicle dynamics simulation allows for a satisfactory analysis of handling behavior during vehicle design, provided that the mathematical model is well-described and accurately represents the vehicle's real response. The motions tracked by the model determine its degrees of freedom (DOF), and the interactions among various vehicle components can be characterized as either linear or nonlinear. Several commercial software programs use sophisticated multi-DOF nonlinear models to analyze vehicle behavior with high fidelity. Simplifying the models to fewer degrees of freedom is advantageous for control applications once the variables of interest are covered with the appropriate level of correlation. Additionally, they are well-suited for applications where the control system has limited computational resources and cannot accommodate more complex models.

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Even with computational tools that apply sophisticated techniques for vehicle dynamics simulation, simplified models continue to be widely explored due to their practicality in specific applications. For example, in [1], a vehicle dynamics simulation tool was developed using simplified models, providing an alternative for researchers and developers to expensive commercial software.

Research related to the assembly of vehicle models focuses on parameter sensitivity and the inclusion of new effects. In [2], a 2-DOF linear model is proposed to investigate the influence of design parameters such as the center of gravity position, vehicle mass, tire stiffness, and vehicle speed on its dynamic stability. In [3], linear models describe the vehicle's lateral dynamics in various driving scenarios. Using a nonlinear model, [4] analyzes the skidding mechanism in steady-state and transient states, considering longitudinal, lateral, roll, and yaw motions. [5] performs Monte Carlo sensitivity simulations to study the effects of vehicle parameters such as geometry, tire properties, and suspension parameters on lateral and roll stability using a 7-DOF model. [6] conducts bifurcation analysis of the vehicle-driver system using simplified and complex vehicle and driver models with real-time applications in automotive simulators.

The popularity of various vehicle models for handling analysis is directly related to their applicability in developing control systems. Among the publications, research explores control strategies to enhance vehicle performance, safety, and automation of the systems involved. In [7], an integrated controller is proposed for a vehicle's yaw stability and rollover stability systems using a 3-DOF nonlinear model. [8] proposes an adaptive Electronic Stability Control (ESC) strategy based on a 2-DOF vehicle model that can adjust to changes in vehicle characteristics and road surface conditions. In [9], a higher-level vehicle stability controller based on Model Predictive Control (MPC) is developed, comparing the effects of model complexity by including or excluding vehicle roll motion.

Part of the related works have been comparing and validating simplified vehicle models with simulated and experimental data, while others have focused on developing model-based stability control systems (ESC). In the current scenario, the industry increasingly invests in virtualizing vehicle tests through automotive simulators capable of simulating sophisticated models validated experimentally. However, little emphasis has been added to the ESC project for application in simulators, which combines the operation of the simulator's complex plant model with simplified model-based control.

Therefore, this work, encouraged by the trend in the automotive sector towards the use of vehicle simulators, provides the following contributions: the process of obtaining simplified theoretical models for ESC with adjusted and optimized parameters based on the complex simulator model; the statistical and graphical analysis of the correlation between the simplified models with the complex simulator model; and the exemplification of ESC based on a simplified model applied to the plant of the complex simulator model.

This paper is organized as follows. Section II presents the vehicle model of the VI-CarRealTime software used for simulations. Section III describes the development of theoretical models for correlation. Section IV explains the methods used to evaluate the correlations. Section V presents the results obtained. Finally, Section VI summarizes the conclusions of the study.

II. Theoretical Models

The vehicle dynamics theory allows the construction of models with different degrees of freedom according to the desired motions for modeling. A vehicle model resembling the real system can have multiple degrees of freedom. However, simplification can be useful for a more practical analysis of specific dynamics while maintaining a realistic response.

For constructing models based on physical principles, the coordinate system adopted follows the SAE J670 standard [10], with the axis orientations shown in Fig. 1. Similarly, the Tab. I presents the notation of variables used for the following equations in the subsequent sections.

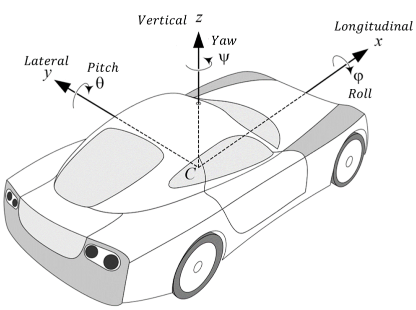

Fig. 1. Coordinates adopted for vehicle orientation according to the SAE J670 standard with the z-axis pointing upwards. (Adapted from [11]).

TABLE I
Variables and Parameters Nomenclature

m	Total mass	κ	Tire longitudinal slip
m_s	Sprung mass	α	Tire slip angle
a	Front axle to CG distance	ω	Wheel angular speed
b	Rear axle to CG distance	v_w	Wheel linear speed
h	Rolling axis to CG height	u	Longitudinal speed
g	Gravity acceleration	v	Lateral speed
c_D	Aerodynamic drag coefficient	β	Side-slip angle
R	Effective tire radius	r	Yaw rate
K_φ	Total roll stiffness	p	Roll rate
C_φ	Total roll damping	φ	Roll angle
I_{xx}	Roll inertia	F_r	Aerodynamics drag force
I_{zz}	Yaw inertia	F_x	Longitudinal force
I_{xz}	Roll yaw combined inertia	F_y	Lateral force
I_w	Wheel rolling inertia	F_z	Vertical force
δ_f	Front steering angle	M_x	Roll moment
T	Wheel torque input	M_z	Yaw moment

A. Body Dynamics

The mathematical modeling of chassis motion is based on the bicycle model, where a single wheel represents both the right and left wheels of the car, as shown in Fig. 2. The equations can be derived using Newton's Laws or the Lagrange Method.

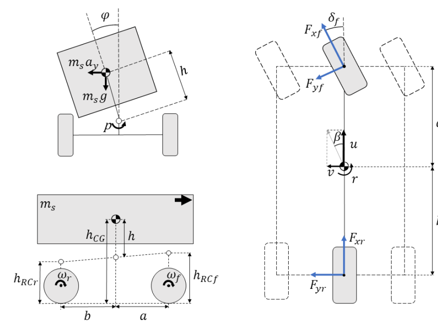

Fig. 2. Vehicle dynamics bicycle model.

In this study, the chassis' longitudinal, lateral, yaw, and roll dynamics will be modeled since they are considered essential for maneuverability and control applications such as ESC. The Eq. 1 shows the resulting equations for each of these motions [4], [12].

$$\sum F_x = m\dot{u} - mvr + m_s hpr \tag{1a}$$
$$\sum F_y = m\dot{v} + mur - m_s h\dot{p} \tag{1b}$$
$$\sum M_z = I_{zz}\dot{r} - I_{xz}\dot{p} \tag{1c}$$
$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r} - m_s h(\dot{v} + ur) \tag{1d}$$

Similarly, the equations for the external forces that complete the equality are shown in Eq. 2, and they mainly consist of the components of the forces generated on the tire and the aerodynamic resistance.

$$\sum F_x = F_{xf}\cos\delta_f - F_{yf}\sin\delta_f + F_{xr} - F_r \tag{2a}$$
$$\sum F_y = F_{xf}\sin\delta_f + F_{yf}\cos\delta_f + F_{yr} \tag{2b}$$
$$\sum M_z = a\left(F_{xf}\sin\delta_f + F_{yf}\cos\delta_f\right) - bF_{yr} \tag{2c}$$
$$\sum M_x = (m_s gh - K_\varphi)\varphi - C_\varphi p \tag{2d}$$

B. Wheel Dynamics

The wheel dynamics are particularly relevant for studying the longitudinal dynamics of the chassis since the input torque to the wheels, provided by the vehicle powertrain, is included in its modeling, as shown in Eq. 3. This equation is adopted for the front and rear axle of the bicycle model, and it considers the dynamic tire parameters and the longitudinal forces generated on the tire.

$$I_w\dot{\omega}_i = T_i - F_{xi}R \tag{3}$$

The wheel speed resulting from the moment in Eq. 3 can differ from the wheel speed due to the vehicle motion, which causes the tire to slip. Regarding the speeds due to vehicle motion, there are both lateral and longitudinal components, as shown in Eq. 4.

$$v_{wxf} = u\cos(\delta_f) + (v + ar)\sin(\delta_f) \tag{4a}$$
$$v_{wyf} = u\sin(\delta_f) + (v + ar)\cos(\delta_f) \tag{4b}$$
$$v_{wxr} = u \tag{4c}$$
$$v_{wyr} = (v - br) \tag{4d}$$

C. Tire Model

The tire model construction especially aims to find the generated forces that impact the vehicle's motion. This requires an understanding of the physical functioning of the tire.

In this context, the tire-road interaction causes deformations in the tire that are directly related to its elasticity. When the resulting speed at the wheel center is not in its median plane, the contact patch undergoes a distortion that defines a slip angle (α) [13]. Similarly, horizontal deflections connected to the wheel's rotational motion induce a longitudinal slip [12]. The Fig. 3 describes the effects of generating forces and moments on tires in detail.

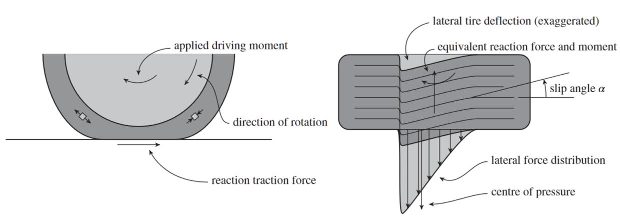

Fig. 3. Tire phenomena that define longitudinal slip and slip angle for force generation (adapted from [14]).

Mathematically, the tire slip angle and longitudinal slip are given in Eq. 5 as a function of the resulting wheel speeds due to wheel rotation and vehicle motion.

$$\kappa_i = \frac{\omega_i R - v_{wxi}}{\max(\omega_i R, v_{wxi})} \tag{5a}$$
$$\alpha_i = \arctan\frac{v_{wyi}}{v_{wxi}} \tag{5b}$$

However, this definition of slip does not consider the responses in time with varying inputs, which suggests using a transient model. With the change in steering angle, the tire must roll for half a turn or more for the lateral deflection and force to build up. This distance is commonly called the "relaxation length" represented by σ_y [15]. The linear differential equation for the transient slip angle α_T is given in Eq. 6 concerning the vehicle speed and the previously calculated instantaneous slip angle (α_i).

$$\dot{\alpha}_{Ti} = -\frac{u}{\sigma - y}\alpha_{Ti} + \frac{u\tan\alpha_i}{\sigma_y} \tag{6}$$

Several mathematical representations relate tire slip to the forces generated on it. Among them, the Magic-Formula proposed by [16] is a semi-empirical method that describes tire behavior for fairly smooth roads up to frequencies of 8 Hz, and it is applicable for all generic vehicle handling and stability simulations [17]. The Eq. 7 shows the calculation for the tire lateral or longitudinal pure force that is part of the Magic-Formula where B_i is the stiffness factor, C_i is the shape factor, D_i is the peak factor, E_i is the curvature factor, S_{hi} is the horizontal offset and S_{vi} is the vertical offset [18].

$$F_i = D_i sin(C_i \tan^{-1}(B_i\phi_i)) + S_{vi} \tag{7a}$$
$$\phi_i = (1 - E_i)(s_i + S_{hi}) + \frac{E_i}{B_i}\tan^{-1}(B_i(\alpha + S_{hi})) \tag{7b}$$
$$s_i = \alpha, \kappa \tag{7c}$$
$$i = x, y \tag{7d}$$

The full Magic-Formula extension is present in manuals that apply the equation due to the desired model version. The manual [17] provides the equations for the tire model PAC2002 with the parameters declared in the *.tir* file of the tire of the vehicle subject of this study. For simplification purposes, only the lateral and longitudinal forces, along with the combined effect, will be calculated through the manual.

D. Mathematical model

Finally, the vehicle models built adopt different degrees of freedom according to the chassis motions represented in the bicycle model, as shown in Tab. II.

TABLE II
DYNAMICS INCLUDED FOR EACH THEORETICAL MODEL CONSTRUCTED AS A FUNCTION OF THEIR DEGREES OF FREEDOM (DOF)

	Longitudinal Dynamic	**Rolll Dynamic**	**Lateral Dynamic**	**Yaw Dynamic**
2-DOF			x	x
3-DOF		x	x	x
6-DOF	x	x	x	x

The sensitivity of each model can be evaluated and compared for the same driving scenario, which reveals whether the complexity of the model is decisive for capturing the investigated effects. Thus, it is important to highlight the following considerations regarding these models:

- The center of the vehicle dynamic coordinate system is fixed at the same position as its CG;
- Regardless of the suspension travel, the vehicle only moves parallel to the track, which is uniform and flat;
- Only the 6-DOF vehicle model that considers the longitudinal dynamics will receive the input signal of the resulting torque on each axle due to traction and braking;
- The 2-DOF and 3-DOF models that have no longitudinal dynamics treat the initial vehicle speed at the maneuver as constant;
- The effect of the steering system is neglected, and the front wheel steering angle is used as a direct input;

- Any load transfers to the wheels are neglected;
- The mechanical properties, inflation pressure, and camber angle of the tires do not change;
- Lateral and longitudinal forces in compound effect tires are based on the Magic-Formula for the PAC2002 version;
- Null vertical and horizontal offset coefficients;
- Slip angle transient considered for 0.562 of relaxation length.

The schematic in Fig. 4 shows the flow of the calculations developed in the theoretical models built in Matlab-Simulink with the equations presented and the input data from the VI-CarRealTime model for output correlation purposes.

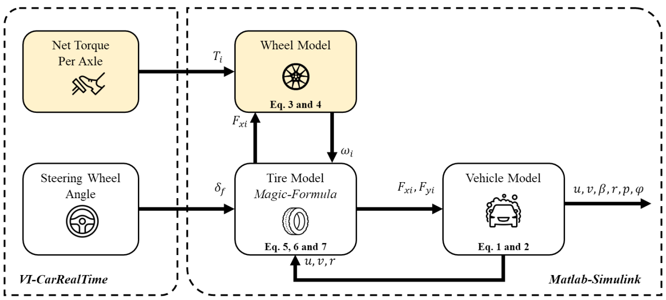

Fig. 4. Simulation diagram of the theoretical models with the equations presented in Matlab/Simulink where the inputs come from the maneuver built in VI-CarRealTime. The blocks highlighted in yellow correspond only to the 6-DOF model, which includes the longitudinal dynamics.

III. VI-CarRealTime Model

Currently, there are established software applications in the industry for vehicle dynamics simulation and analysis that exhibit high fidelity to the real vehicle response. The VI-CarRealTime software used in this study is developed by VI-Grade for vehicle dynamics simulation using a complex 14-DOF model as described in its manual [19].

This software enables the analysis of various driving scenarios. It considers a range of parameters from different vehicle subsystems, including chassis, powertrain, brakes, suspension, steering, tires, and electronic control systems. The selected object of study for this work belongs to the VI-CarRealTime vehicle model library, and it represents a sedan-type vehicle. The desired maneuvers for simulation are also built within the environment of this software.

IV. Correlation Analysis

The correlations in this study compare the handling responses between the VI-CarRealTime complex model and the theoretical models built. For this purpose, the model parameter tuning and the statistical analysis of the response are important processes to perform the correlations. Thus, the sedan vehicle from the VI-CarRealTime model library has its parameters imported to the theoretical models, as shown in Tab. III.

However, some parameters required for the theoretical models are derived from the whole suspension subsystem and must be estimated from the vehicle response. Therefore, this study applies two techniques to find the values of these parameters: the characterization process and optimization methods.

TABLE III
Parameters of the Vehicle Under Analysis

Parameter	Unity	Value
m	kg	1986.6
m_s	kg	1760.3
a	m	1.332
b	m	1.541
I_{xx}	kg m²	527.9
I_{zz}	kg m²	2943.6
I_{xz}	kg m²	0.059
I_w	kg m²	1.389

A. Characterization Process

The first technique, known as characterization, analyzes the responses of the fast transient and the permanent regime from the step steer maneuver performed in VI-CarRealTime. The main purpose is to mathematically estimate the remaining constant parameters for the theoretical models by analyzing their variations in time.

Among the required parameters, the rolling axis height to CG (h) and the effective tire radius (R) are defined from the average values in time. The rolling stiffness coefficient (K_φ) is defined by its value estimated from the equations of motion in the steady state, while the rolling damping coefficient (C_φ) is defined similarly in the transient state.

B. Optimization Methods

Even after characterizing the parameters, it is expected that the correlation of the roll variables may be unsatisfactory due to the influence of the vehicle's suspension compliances defined only by the two parameters K_φ and C_φ. In this case, optimizing these parameters is proposed for the same maneuver as the characterization process and with the initial values obtained from it. The objective function is based on the Root Mean Square Error (RMSE) minimization of the roll responses directly impacted by the selected parameters.

In order to select the optimization method, two algorithms available as Matlab functions are compared to choose the one with the lowest RMSE value. The first algorithm applies the Simplex Nelder-Mead method, which performs reflection, expansion, contraction, and shrinkage operations to find the minimum around the initial value through a convex polygon of a dimension equivalent to the number of variables, but it requires a well-estimated initial value for the result to be consistent [20]. The second algorithm applies the Interior-Point method, which uses the interior barrier approach to transform the problem into an unconstrained problem and deals well with convex optimization problems [20].

C. Statistical Validation

Finally, with the parameters set, simulations are performed to analyze and correlate emergency maneuverability maneuvers.

The first maneuver performed is called Sine With Dwell, where the vehicle is subjected to a sinusoidal pattern of steering at 0.7 Hz frequency with 500 ms delay in the second peak amplitude as shown in Fig. 5, the steering is initiated with the vehicle coasting in high gear and at a speed of 80 ± 2 km/h [21]. This maneuver is widely used for calibration and even homologation of the stability control system (ESC).

The second maneuver, called Double Lane Change, is simulated in a closed loop with the VI-CarRealTime driver model, and it consists of the vehicle quickly changing lanes according to Fig. 5 as if it were a fast overtaking or obstacle avoidance. This maneuver also makes it possible to evaluate the vehicle's stability and safety under extreme conditions and aid in developing and calibrating ESC.

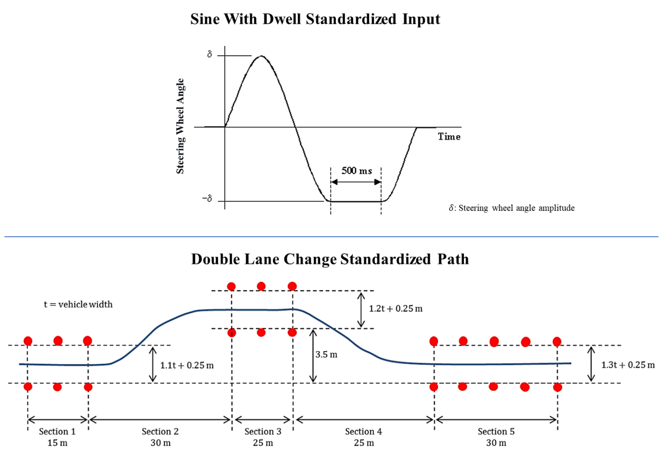

Fig. 5. Standardized path for Double Lane Change maneuver according to ISO 3888-1 [19]. Standardized steering wheel angle input for Sine With Dweel maneuver [21].

After the maneuvers are performed, the results of the state variables and the total forces generated in the bicycle model are post-processed to present the overlapping signals from the different simulated models graphically. In addition, statistical analysis using the R^2 statistics statistic metric provides a fit measurement on a scale between 0 and 1, which will be presented as a percentage in the results section. However, determining what would be a *good* R^2 value is challenging and depends on the application, especially the linearity of the problem [22]. Both graphical and statistical evaluations contribute to determining the correlation quality between each theoretical model and the reference VI-CarRealTime model.

V. Results

The first stage of the results arises from post-processing the step steer maneuver in VI-CarRealTime for estimating the sedan vehicle parameters by the characterization process. The Fig. 6 shows the plots with the steering wheel angle input signal and the equivalent signals varying the desired parameters. It is noticed that steering ratio and roll stiffness were observed in the steady state region of the maneuver, while roll damping was observed in the transient state range.

However, the characterization of the theoretical models causes uncertainty in the value of the roll parameters, as predicted in the previous section. For this reason, optimizing the roll damping and roll stiffness parameters is proposed to minimize the roll rate and roll angle errors between the theoretical model and the reference. To do so, only the theoretical 3-DOF model was selected, and two optimization methods were compared, as shown in Fig. 7 for the same step steer maneuver from the characterization process. Both methods provided similar results with close RMSE values at the end of the optimization.

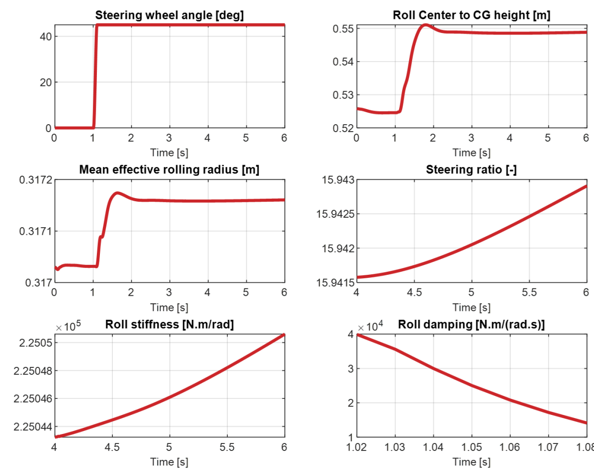

Fig. 6. Characterization process for parameter estimation from the step steer maneuver.

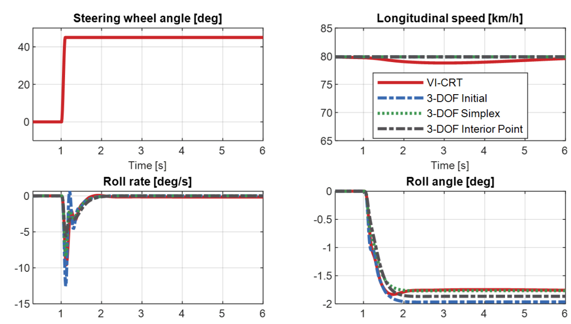

Fig. 7. Comparison of the initial 3-DOF theoretical model correlation and those optimized by the Simplex and the Interior Point methods.

Thus, the optimized response of the Simplex method was selected, which proved to be slightly better. Finally, Tab. IV shows the constant values of the parameters estimated in the characterization and the optimized parameters, which will be used for all the other simulations of the theoretical models.

TABLE IV
Parameters Defined after Characterization and Optimization Procedures

Parameters	Unity	Estimated value	Optimized value
τ	-	15.9	-
h	m	0.546	-
R	m	0.317	-
K_φ	N m/rad	225046	238014
C_φ	N m/(rad/s)	2608	2204

The first result for the Sine With Dwell maneuver is shown in the graphs of Fig. 8 and in the statistical analysis in Tab. V.

Initially, it can be seen that the 2-DOF and 3-DOF models keep the initial speed constant while the 6-DOF model can represent the longitudinal motion, which also impacts the correlation of the side-slip angle. This justifies the R^2 values of over 90% between the 6-DOF model and the VI-CarRealTime (VI-CRT) reference, reflecting a high level of correlation that can also be seen graphically. On the other hand, the remaining 2-DOF and 3-DOF models have an average level of side-slip angle correlation, with R^2 values close to 75%. Even so, they are coherent in terms of their graphical shape and still keep a high level of correlation with the other monitored variables.

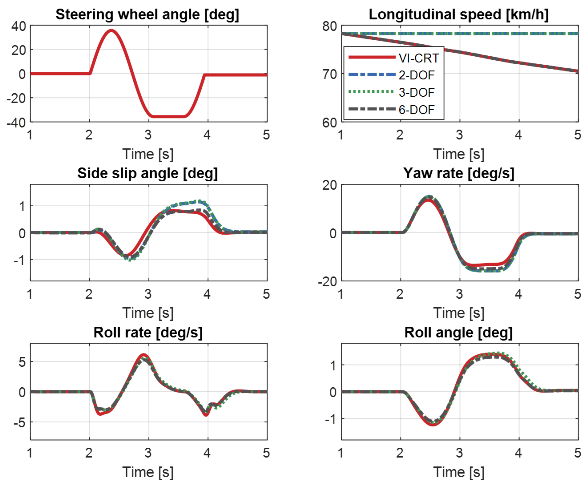

Fig. 8. Sine With Dwell maneuver correlation graphs.

TABLE V
SINE WITH DWELL MANEUVER CORRELATION STATISTICS

R^2 Statistics	2-DOF	3-DOF	6-DOF
Side-slip angle	78.5%	75.7%	91.2%
Yaw rate	96.4%	96.4%	98.4%
Roll rate	-	97.9%	98.2%
Roll angle	-	98.8%	98.9%
Longitudinal speed	-	-	99.9%

The second result for the Double Lane Change maneuver shown in the graphs of Fig. 9 and in the statistical analysis in Tab. VI has similar observations to the previous one. However, the maneuver performed with constant speed control on the VI-CRT reference improves the side-slip angle correlation for the 2-DOF and 3-DOF models graphically and statistically, with R^2 values close to 90%. Concerning the other variables analyzed, high correlation values can be seen for all models, except for longitudinal speed for the 6-DOF model. Although this signal is very close to the reference graph, the error caused by the increasing amplitude offset with time affects the R^2 value, suggesting a low level of correlation but not enough to affect the other variables. In these situations, the rigor of the R^2 metric and the importance of graphical analysis in assessing the correlation's quality become clear.

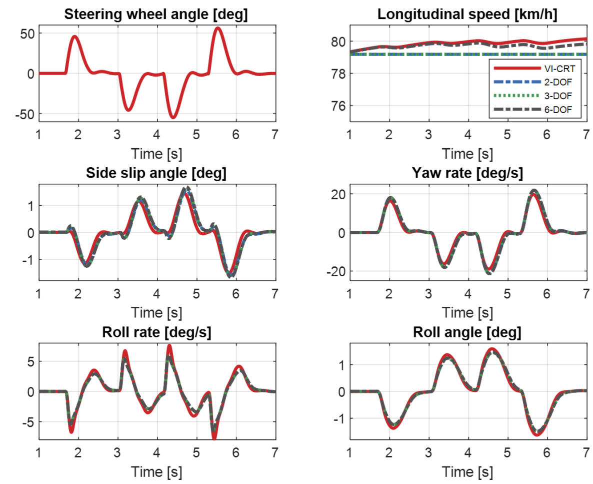

Fig. 9. Double Lane Change maneuver correlation graphs.

TABLE VI
DOUBLE LANE CHANGE MANEUVER CORRELATION STATISTICS

R^2 Statistics	2-DOF	3-DOF	6-DOF
Side-slip angle	89.7%	88.1%	86.8%
Yaw rate	97.1%	96.9%	96.6%
Roll rate	-	96.5%	96.4%
Roll angle	-	98.9%	98.9%
Longitudinal speed	-	-	31.7%

Finally, the fate of the correlated theoretical models is exemplified in the graphs of Fig 10, which compare the simulation results of the VI-CRT model for the Sine With Dwell maneuver without control and with the high-level ESC based on the 3-DOF model. These graphs also show the desired side-slip angle and yaw rate signals based on those calculated for the steady state and the yaw-correcting ESC moment applied to the vehicle for high-level control. The control method is based on the Linear Quadratic Regulator (LQR) for calculating the optimum state feedback gain that applies the corrective moment.

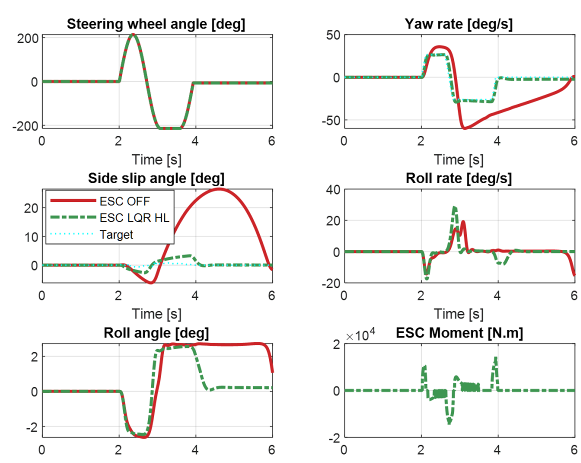

Fig. 10. Sine With Dwell simulated on VI-CarRealTime without ESC (ESC OFF) and with ESC high-level 3-DOF model-based LQR (ESC LQR HL).

VI. Conclusion

This paper explored the theoretical 2, 3 and 6 degrees of freedom models based on the physical modeling of vehicle dynamics and their performance compared to the complex model of VI-CarRealTime. The constant parameters required for these models to operate are provided by the selected vehicle information and simulated maneuvers using the characterization and optimization techniques presented. In terms of results, the correlations analyzed prove the potential of simplified theoretical models to represent the dynamics involved in vehicle stability control despite their limitations due to neglected effects. The exemplified control scenario reveals the main application of interest where ESC based on simplified models is implemented in the plant of the VI-CarRealTime complex model used in automotive simulators.

Therefore, when defined using the presented procedure, simplified models can achieve responses as accurately as complex multi-DOF models from dedicated software. Their advantages include the ease and quick tracking of impact parameters for an interested dynamic response, as well as their suitability for ESC applications, as exemplified in the results. The models presented motivate future works, especially the modeling or calibration of different ESC strategies based on a model with the possibility of subjective evaluation in an automotive dynamic simulator or incorporation into Hardware in the Loop systems.

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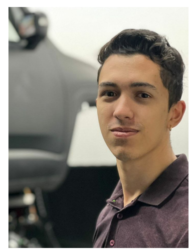

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