Application of Optimization Techniques for Generating Trajectories and Adjusting the Controller Gains of a Hydraulic Servo-Positioner using the Firefly Metaheuristic Algorithm

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***Abstract*—This work evaluates the influence of the reference trajectory on the tracking error in a servo-positioner control system. Thus, the objective is to improve the ideal tuning of the gains of a controller applied to the tracking of the positional trajectory of a hydraulic actuator through the physical characteristics of the plant and the trajectory. The applied controller uses a cascade strategy and consists of dividing the mathematical model into two interconnected subsystems, one hydraulic and the other mechanical, applying specific control strategies to each subsystem. The proposed methodology is implemented using the Firefly Metaheuristic Algorithm (FMA). The first stage consists of generating the 5th order optimal trajectories by means of b-splines functions, in which they must minimize the acceleration along the actuator's path, considering speed and flow restrictions related to the hydraulic servo-position. The second step consists in determining the effective value of the error during the execution of the trajectory and the respective gains applied to the model. The results show that this strategy proved to be useful for obtaining adequate trajectories and gains in plants with significant non-linearities, because the trajectory error was 27% lower than the empirical adjustment method of gains compared in this study.**

***Keywords*— Firefly Metaheuristic Algorithm, Cascade Control, Hydraulic Servo Positioner, Optimization, Trajectory Planning**

I. INTRODUCTION AND PROBLEM STATEMENT

The hydraulic actuators has characteristics that hinder their feedback control in high performance applications [1], [2]. The mathematical model of this system shows important non-linearities [2], such as the relationship between the control variable and the oil flow, the frictional force between the hydraulic cylinder and the piston, the internal system leaks, the dead zone and saturation, which are present in the control valves.

Many control algorithms have been developed seeking to increase the performance in the control of hydraulic actuators and, with this, expand the field of applications of these systems. Among the control techniques, we can highlight those based on feedback linearization [3], neural networks [4], backstepping [5], among others. Another control strategy that can be used in this area is that of cascade control [6]–[8]. This type of controller is characterized by allowing the interpretation of the hydraulic plant as subdivided into a mechanical and a hydraulic subsystem. There are different types of approach to the cascade strategy, such as, for example, controllers with fixed gains [8], adaptive [6] and using neural networks [9], [10].

In order to obtain an adequate performance regard to the position path error in hydraulic actuators, it is essential to make an adequate adjustment of the gains on the controllers [6]. Some adjustment techniques for automatic tuning of controller gains are presented in [11]–[13], which propose a method for tuning the gains of a PID controller (Proportional-Integral-Derivative) through intelligent systems. Although this approach is efficient, it has limitations due to the position error when using a PID controller. This is due to the presence of undamped poles in the dynamics of the hydraulic actuators (open loop) and the non-linearities of the system [14]. There are also studies that present strategies for adjusting the gains of controllers that use non-linear control algorithms. In the case of cascade controllers, for example, there are works that apply heuristic tuning methods [15], [16], automatic tuning methods [6], methods based on metaheuristic algorithms [17], among others.

The adjustments of the gains are fundamental to obtain desirable performances in the use of the controllers, however, the characteristics of the reference trajectory applied to the plant can also directly influence the result of the tracking error. To apply some control strategies, as in [11] and [18], the paths used in cascade controllers must be smooth and continuous until their 3rd derivative, when the system requires position, speed and acceleration references. In [17], for example, sinusoidal and 5th order polynomial trajectories were used to determine the optimal gains of a cascade controller applied to a hydraulic servo-positioner.

Heuristic algorithms are techniques that are increasingly being used to solve problems related to the adjustment of gains and the generation of optimal trajectories for the most different types of industrial controllers. According to [19], these

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algorithms are characterized by applying a simple set of procedures to obtain solutions, even if it is not the optimal global solution, simply and quickly. This work proposes the use of the Firefly Metaheuristic Algorithm (FMA), which present similar performances to deterministic optimization methods. The FMA is characterized by its effectiveness in the search for a local optimum and a global optimum in a synchronous way. These algorithms allow the adjustment of parameters such as the variation of attractiveness and randomization adjustment, which are presented in more detail in [20]. The choice of this algorithm is based on the fact that the physical system and the proposed controller have a non-linear behavior, and these methods allow to obtain optimal global results [20], [21]. Other works have used different metaheuristic algorithms for generating trajectories and non-linear control. For example, in [22], the Bald Eagle Search (BES) algorithm with dimension learning-based hunting (DLH) was used for research on autonomous vehicles. In [23], a proposal for manipulator robot control using Butterfly Optimization Algorithms (BOA) was presented, with the aim of minimizing the robot trajectory error. In [24] the author employed a non-linear model predictive control (NLMPC) for manipulator robots, and optimized the controller gains using a Neural Network Algorithm (NNA) to compensate for non-linearities and reduce the error in the robot's trajectory tracking.

Non-linear control algorithms have a satisfactory performance when applied to hydraulic servo-positioners [10]. Recently, some studies have sought to apply intelligent techniques based on heuristic strategies to adjust the gains of this type of controller [17], [25], [26], among others. In this sense, it is possible to observe that there are few approaches that deal with the use of optimized trajectories to improve the performance of these systems.

The contribution of this paper is to propose a method for improving the efficiency of cascade controllers [10], [17] in terms of reducing the error in tracking the trajectory. Since a cascade controller requires a physical system (plant), a specified trajectory, and a method for adjusting the gains, this paper evaluates the impact of the reference trajectory on the tracking error, demonstrating that the optimal adjustment of the gains can be improved by considering both the plant's physical characteristics and the trajectory.

The proposed method for finding the optimal reference trajectory, presented in subsection IV.C, uses the FMA to minimize the derivative of the speed along the actuator's path, while respecting the inequality restrictions related to the actuator's kinematic characteristics and the peculiarities of the hydraulic system, as described in section II. This is crucial for reducing errors resulting from the dynamics of hydraulic systems. In subsection IV.D, the gain adjustment technique is presented. The methodology proposed uses the FMA to minimize the error during the execution of the trajectory in order to obtain the optimal gains. The optimal solution is the best combination of gain values that minimize the error in the system's response while respecting the limits for the cascade controller's gains. The results show that the tracking error decreases significantly, without sacrificing the control signal or the actuators' efforts, compared to the method presented in [10].

The rest of the manuscript is organized as follows: in sections II and III present, respectively, the mathematical model of the hydraulic actuator used and the theoretical aspects of the cascade control strategy. In Section IV, the methodology for optimization the trajectory and gains applied to the cascade controller is presented. In Section V, the simulation results obtained applying the proposed methodology are presented and discussed. Finally, Section V presents the conclusions of this work.

II. The Hydraulic Actuator Model

According to Newton's second law, the dynamic response of the mechanical system is represented by (1).

$$F_H = p_1 A_1 - p_2 A_2 = M\ddot{y} + B\dot{y} \quad (1)$$

where y, $\dot{y}$ and $\ddot{y}$ are the piston position, speed and acceleration. The camera pressures are p_1 and p_2. A_1 and A_2 are the piston areas. M represents the mass of the piston and charge. F_H is the hydraulic force of the piston, resulting from the pressures p_1 and p_2 and the viscous friction coefficient B, which, multiplied by the speed, represents the frictional force between the piston and the cylinder F_A. V_1 and V_2 are the camera's volumes. The signal control is u, P_o and P_S are the fluid pressure, and finally, Q_1 and Q_2 are the fluid flow. Fig.1 shows the hydraulic actuator used in this work.

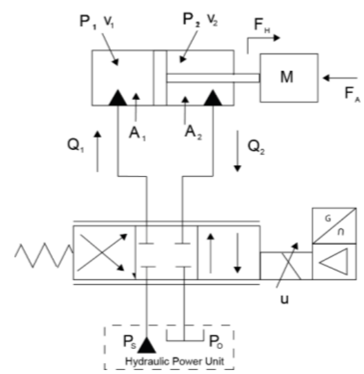

Fig. 1. The hydraulic system: double-acting cylinder and 5/2-way proportional valve [2].

According to [14], the mathematical model of its dynamic behavior can be obtained by applying Newton's second law and the fluid flow continuity equation. Now, combined (1) with (2), explains the derivative of hydraulic force and facilitates the application of the control scheme proposed in section III.

$$\dot{F}_H = (A_1 f_1 K_{v1} g_1 + A_2 f_2 K_{v2} g_2)u - ({A_1}^2 f_1 + {A_2}^2 f_2)\dot{y} \quad (2)$$

More information about the hydraulic actuator model sees in [15].

III. The Cascade Controller

The cascade control strategy applied to hydraulic actuators is based on the mathematical model of the hydraulic actuator and considering two interconnected subsystems (Fig.2), a hydraulic subsystem and another mechanical subsystem [2], [14]. This form of control can also be described as follows:

(i) Calculate a control law F_{Hd} (desired hydraulic force) for the mechanical subsystem, so that the output y (position) follows the trajectory y_d (desired position) with the least possible error.

(ii) Calculate a control law u for the hydraulic subsystem, so that the hydraulic force F_H follows the reference F_{Hd} provided by the previous control law, with the least possible error.

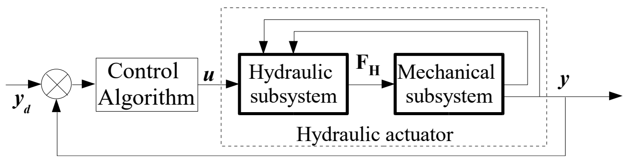

Fig. 2. Schematic diagram of Cascade Control [2].

As in [14], where a mechanical subsystem expressed by (1) is proposed, the control law is based on the Computed Torque law, used in [2] and [14], where the desired hydraulic force is expressed by (3).

$$F_{Hd} = M\ddot{y}_r - K_d z + B\dot{y}_r \tag{3}$$

where $\ddot{y}_r$ and z are defined by (4).

$$\ddot{y}_r = \ddot{y}_d - \lambda\dot{\tilde{y}}, \qquad y_r = \dot{y}_d - \lambda\tilde{y}, \qquad \tilde{y} = y - y_d, \\ z = \dot{y} - \dot{y}_r = \tilde{y} + \lambda\tilde{y} \tag{4}$$

where K_d and λ are the controller gains, $\ddot{y}_r$ is a reference acceleration and z is the tracking error. Applying this control law, the expression in closed loop is obtained in (5).

$$\tilde{F}_H = M\dot{z} + (K_d + B)z \tag{5}$$

where the force error is defined in (6).

$$\tilde{F}_H = F_H - F_{Hd} \tag{6}$$

In the hydraulic subsystem, expressed by (3), the feedback linearization strategy is used as a control law, where the non-linearities of the system are cancelled when all parameter values are known. The feedback linearization strategy allows the cancellation of non-linearities and the imposition of linear dynamics and can be applied in cases of non-linear systems described in the companion form. A system is said in the companion form if its dynamics is represented by (7).

$$x^{(n)} = f(\boldsymbol{x}) + b(\boldsymbol{x})u \tag{7}$$

where u is a scalar control input, x is the scalar output of interest, $\boldsymbol{x}$ is the state vector, $f(\boldsymbol{x})$ and $b(\boldsymbol{x})$ are the nonlinear state functions. Using this representation and choosing the control input, we obtain (8).

$$u = \frac{1}{b}(v - f) \tag{8}$$

Using (3) and (8), we have the control law in (9).

$$u = \left(\dot{F}_{Hd} - K_p\tilde{F}_H + ({A_1}^2 f_1 + {A_2}^2 f_2)\dot{y}\right)\frac{1}{(A_1 f_1 K_{v1} g_1 + A_2 f_2 K_{v2} g_2} \tag{9}$$

Through this law and when all parameters of the model are known, the dynamics of the closed-loop system is expressed by (10).

$$\dot{\tilde{F}}_H = -K_p\tilde{F}_H \tag{10}$$

In this case, asymptotic convergence of the position error to zero occurs, and the time is dependent on the value of the K_d, λ and K_p gains [2], [14].

IV. Optimization of the Trajectory and Gains Applied to the Cascade Controller

The cascade controller parameters are optimized in two steps through an offline procedure. The optimization techniques presented in subsections IV.C e IV.D were applied using the Firefly metaheuristic algorithm (FMA). To obtain the optimal trajectory interpolated by 5th degree b-splines functions, it is first necessary to establish the intermediate points. The proposed method is based on determining the trajectory that minimizes the acceleration during the movement of the actuator, considering the physical and kinematic restrictions of the system. A trajectory with high accelerations leads to high actuator efforts and, consequently, a significant increase in tracking error due to saturation in the system. The controller gain optimization technique is similar to the approach presented in [17], where the ideal values of the k_d, k_p and λ gains are determined for a situation in which the trajectory is predetermined. The main contribution of this work is the development of a strategy that improves the dynamic response of a closed loop system. This strategy will be applied to a hydraulic actuator, by optimizing the reference trajectory and the controller's gains. Thus, a strategy will be presented to determine the optimal trajectory of reference, considering the characteristics of the plant (1st stage of optimization), to later determine the optimal adjustment of gains. The results of the optimizations are later used in simulations with the purpose of evaluating the performance of the closed loop system. The result of the proposed strategy is evaluated by means of a simulation of the displacement of the piston in the entire path of the servo-positioner, considering the two stages of optimization proposed.

A. The Firefly Metaheuristic Algorithm

The FMA was elaborated by [20] based on observations of flashing behavior of fireflies, so that the bioluminescence effect is the essence of the method. Specifically in the case of fireflies, the grouping for these insects is based in the rhythmic flash, the rate of flashing and the amount of time it remains on. Also, in nature, distance plays an important role since it may impair the visibility of other fireflies, and on optimization problems, the cost function is evaluated as the brightness of the fireflies.

According to [20], the FMA algorithm was developed for optimization problems based on three ideal rules: all fireflies are unisex; the attractiveness decrease as the distance increases; and a random search is used in the case of no existence of brighter firefly.

1) *Attractiveness, absorption and light intensity*

According to [20] and presented in [27] and [28], the formulation of the attractiveness and the variation of light intensity are two important issues that should be addressed. As previously said, the objective function $f(\mathbf{x}^i)$ is associated to the brightness $I(\mathbf{x}^i)$ where $\mathbf{x}^i = (x_1, x_2, \ldots, x_d)^T$ is a tentative solution vector. So, $I(\mathbf{x}^i) \propto f(\mathbf{x}^i)$. The distance impairs the visibility so the attractiveness β is relative. Thus, the distance r_{ij} between firefly *i* and firefly *j* will affect this attractiveness. The attractiveness can be set to vary with the degree of light absorption (γ).

For fixed light absorption coefficient γ, the light intensity *I* may be formulated as follows [20] and [28], as show in (11).

$$I(r_{ij}) = I_0 \exp\left(-\gamma\, r_{ij}^2\right) \tag{11}$$

where I_0 means the original light intensity. The attractiveness β (light intensity for the neighbor fireflies) of a firefly in this paper is modelled as in (12).

$$\beta(r_{ij}) = \beta_0 \exp\left(-\gamma\, r_{ij}^2\right) \tag{12}$$

where β_0 means the attractiveness at $r_{ij} = 0$. For a fixed γ, the attractiveness $\beta \to \beta_0$ when $r_{ij} \to 0$ and conversely if $r_{ij} \to \infty$ then $\beta \to 0$. The way this variation happens can be calibrated beforehand based on the values of the cost function and the behavior of the algorithm. This is one of the metaheuristic parameters that should the defined by the user. The position update of firefly i attracted to a brighter one j is defined by (13).

$$\mathbf{x}^i = \beta_0 \exp\left(-\gamma\, r_{ij}^2\right) + (\mathbf{x}^i - \mathbf{x}^j)\alpha\, \boldsymbol{\varepsilon}^i \tag{13}$$

where the second term is due to the attraction and the third term $\alpha\, \boldsymbol{\varepsilon}^i$ is a randomization term composed by a vector of random numbers (ε^i). According to [27], [28] these random number are meant to perturb the natural firefly movements. They are generated from a uniform distribution [-1,1] and the randomization parameter α is used to decrease its influence along iterations. In this paper, it is used alpha values updated along iterations *i* (for all examples) as $\alpha^{(i)}$=0.99 $\alpha^{(i-1)}$. The general idea for Eq. (13) is to perform a random search biased towards brighter fireflies.

The parameter γ characterizes the variation of the attractiveness and determines the speed of the convergence and efficacy of FMA in finding a global optimum. A very similar technique is used by Simulated Annealing to improve the convergence rate. As indicated by [20], γ ∈ [0,∞). This parameter typically varies from 0.1 to 10 for normalized cost functions. Fig. 3 shows a pseudo-code for the firefly algorithm in minimization problems.

Define objective function $f(\mathbf{x}^i)$ where $\mathbf{x}^i = (x_1, x_2, \ldots, x_d)^T$ and *d*= no. of design variables
Generate an initial population of fireflies randomly $\mathbf{x}^i$ *i*=1, 2,…, *n*, where *n*=no. of fireflies, ranked in ascending order.
Light intensity *I* at $\mathbf{x}^i$ is determined by $f(\mathbf{x}^i)$ for all fireflies.
Define the light absorption coefficient γ based on the Γ value of the initial population
While *t* < maximum number of generations or convergence criteria are met
  For *i*=1 to *n*
    For *j*=1 to n
      If ($I^i > I^j$), move *i* towards *j*
        Calculate the distance $r_{ij} = |\mathbf{x}^i - \mathbf{x}^j|$
        Calculate $\beta(r_{ij}) = \beta_0 \exp(-\gamma r_{ij}^2)$
        Update design variables $\mathbf{x}^i = \beta_0 \exp(-\gamma r_{ij}^2) + (\mathbf{x}^i - \mathbf{x}^j)\alpha\, \boldsymbol{\varepsilon}^i$
        Update $I^i = f(\mathbf{x}^i)$
      End if
    End For *j*
  End For *i*
  Rank fireflies in ascending order and find the best firefly so far.
End While
Post process results

Fig. 3. FMA pseudo-code. Adapted from [10] and [28].

B. *Definition of the Ttrajectory Using Fifth-Order B-Splines*

Considering that the cascade control method applied to the hydraulic actuator was based on the calculated torque strategy, which, according to [29], requires that the trajectory to be used has continuity in the position function and its first three derivatives, in this work, we chose to use 5th degree trajectories. The choice for 5º b-spline functions is because they are consolidated techniques for generating trajectories, with equations already known in the literature for implementation in optimization algorithms. According to [30] and [31], to interpolate *n + 1* points, the b-spline function can be defined as (14).

$$Q_k = C(u_k) = \sum_{i=1}^{n} N_{i,p}(u_k) P_i \tag{14}$$

where P_i (*i = 0.1, ……, n + p + 1*) are the control points to be determined, $N_{i,p}(u)$ are the base functions and u_k is the value of the node corresponding to the intermediate point Q_k of the trajectory.

The basic functions of b-splines are determined by the De Boor recurrence method, which is useful for computational implementations [31]. The *i-th* base function of the degree *p* (or equivalently order *p + 1*) b-spline is defined according (15) and (16).

$$N_{i,0}(u) = \begin{cases} 1, & u_i \le u \le u_{i+1} \\ 0, & another\ case \end{cases} \tag{15}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{16}$$

$$with \ p > 0$$

From the obtained control points, which are determined through (15), the b-spline function is calculated, in which the resulting curve will pass through all the points Q_k. More information about the formulations of the b-splines functions can be found in [30] and [31].

C. Generation of the Optimal Reference Trajectory

Once the intermediate points of the actuator are known, the proposed method is based on determining a path that is continuous in its derivatives of position, speed and acceleration.

The objective function aims to minimize the derivative of the speed along the path of the actuator, with small time increments ($\Delta t = 0.1$s), as (17).

$$F_{obj} = \left| \frac{dv(\Delta t)}{dt} \right| \tag{17}$$

The inequality restrictions are associated with the kinematic characteristics of the actuator and the particularities of the hydraulic system, presented in section II. The kinematic restrictions are based on the limitations of the maximum values of speed and acceleration, defined by the actuator manufacturer, as presented in (18) and (19).

$$|q(t)| \leq P_C \tag{18}$$

$$|\dot{q}(t)| \leq V_C \tag{19}$$

where $q(t)$ and $\dot{q}(t)$ are the position function and its derivative (position and speed in the actuator along the path), P_C is the limit of displacement of the actuator along the path and V_C is the maximum speed specified actuator manufacturer.

The restrictions on the physical properties of the hydraulic system consist of limiting the flow values and their variation along the path of the actuator. This is because large variations in the flow values can cause a significant increase in the acceleration of the actuator and, consequently, in the error of trajectory tracking. The formulation of this restriction is presented in (20).

$$\begin{gathered} Q_1 < K_{v1} u g_1 \quad g_1 = \begin{cases} \sqrt{p_s - (p_1 + l_1)}, & u \geq 0 \\ \sqrt{p_1 - l_3}, & u < 0 \end{cases} \\ \dot{Q}_1 < Max(\dot{Q}_1) \\ Q_2 < K_{v2} u g_2 \quad g_2 = \begin{cases} \sqrt{p_2 - l_4}, & u \geq 0 \\ \sqrt{p_s - (p_2 + l_2)}, & u < 0 \end{cases} \\ \dot{Q}_2 < Max(\dot{Q}_2) \end{gathered} \tag{20}$$

where K_{v1} and K_{v2} are the volumetric flow gains that characterize each valve orifice; $l_1 \ldots l_4$ are the pressure losses in the hoses; $\dot{Q}_{1,2}$ is the variation of the maximum flow allowed to avoid high accelerations; p_s, p_1, p_2 and u are the variables corresponding to supply pressure, chamber pressure and valve control voltage, respectively.

The design variables (v) are represented by the time interval (h_i) that the actuator needs to move between two successive points in the trajectory, according (21).

$$v = (h_1, \cdots, h_p)^T \tag{21}$$

where h_i is calculated considering the position between two successive points to be interpolated and the maximum speed of the actuator (V_C). The calculation of the lateral restrictions to interpolate p intermediate points is represented by (22).

$$h_i = max\left\{\frac{|q_{i+1} - q_i|}{V_c}\right\} \quad i = 1, \cdots, p-1 \tag{22}$$

D. Gains Adjustment Technique

For the controller to be executed in time, it is necessary to use the optimal trajectories described in the previous section and to tune the values of the most appropriate gains for the operation of the controller. The methodology proposed in this work is based on the approach presented in [17]. The objective function is to minimize the effective value of the error during the execution of the trajectory, according (23).

$$F_{obj} = \sqrt{\frac{1}{T}\int_0^T e(t)^2 dt} \tag{23}$$

where $e(t)$ is the error characterized by the difference between the path of the hydraulic actuator and the desired trajectory, and T is the total time of the path of the actuator in a movement cycle.

The design variables used in this step refer to the gains of the cascade controller, as in (24).

$$V_P = (k_p, k_d, \lambda) \tag{24}$$

After processing the algorithm, the optimal solution consists of the best combination of the set of gain values that minimize the error of the system's response. The lateral constraints are maximum and minimum allowable winnings. Limit values for gains k_p, k_d and λ are presented in (25).

$$\begin{gathered} k_{pmin} \leq k_p \leq k_{pmax} \\ k_{dmin} \leq k_d \leq k_{dmax} \\ \lambda_{min} \leq \lambda \leq \lambda_{max} \end{gathered} \tag{25}$$

V. Results

This chapter seeks to define the ideal parameters of the cascade controller that minimize the error of trajectory tracking for a given movement of the actuator. The analyses performed refer to the implementation of the optimization algorithm in the generation of an optimal reference trajectory and in the tuning of the controller's gains.

In subsection V.A presents the values of the parameters used to generate the optimal trajectory, such as the physical limitations for the path of the hydraulic actuator (maximum flow, maximum speed, position, among others). The aspects evaluated in the trajectory optimization algorithm are: 1) the objective function, checking if this value, in this case, acceleration, meets the operating requirements of the hydraulic actuator; and 2) the kinematic restrictions, analysing whether the position, speed and flow values were violated during the course.

In subsection V.B, it seeks to evaluate which are the adjustable parameters of the cascade controller that minimize the error of following the actuator's trajectory. Knowing the reference trajectory determined in the previous section, this step presents the results of the optimal adjustment of the controller's gains. This method is evaluated by comparing the trajectory tracking errors produced by the original gains of the control algorithm and by the optimal gains.

An experimental analysis was not developed because issues of non-linearity, noise and uncertainties were discussed in the work presented in [10] for the same physical system and controller proposed in this article. The author carried out an experimental analysis to evaluate the trajectory segment error, allowing adjustments to be made to compensate for the non-linearities present in the control model and in the plant. In addition, [10] demonstrated that uncertainties, friction and noise errors did not compromise the experimental results, since the hydraulic forces are much greater than these uncertainty values. That is, the most important variables to evaluate the trajectory segment error are the change in controller gains and the reference trajectory in hydraulic systems. The constraints applied to the optimization problems in subsections V.A and V.B were obtained from the experimental analysis presented in [10]. These constraints were used to limit the physical characteristics of the hydraulic actuator, such as maximum flow, maximum speed, and position, among others. This was done to assess the impact of the reference trajectory on the tracking error and to find the optimal adjustment of the gains for the cascade controller.

A. Analysis of the Optimal Reference Trajectory for the Controller

To show the trajectory generation method and evaluate its results, a case study is presented. In this approach, the actuator travels a set of 18 points established within its operating limits. The key points for the generation were previously defined.

To illustrate the problem proposed in subsection IV.C. the following conditions are considered: 1) The key points used to generate the reference trajectory are Q_i = [0.1000 0.1002 0.1042 0.1146 0.1300 0.1454 0.1557 0.1595 0.1600 0.1600 0.1597 0.1558 0.1454 0.1300 0.1146 0.1042 0.1005 0.1000]. These points were chosen through a sampling of the point-to-point trajectory presented in [10]; 2) The maximum allowable flow rate in the chambers $(Q_{1,2})_{max}$ is $3.2253 \times 10^{-5}\ m^3/s$; 3) The dynamic aspects of the controller, such as force, friction, pressure, temperature, control signal, among others, are not defined as restrictions of the optimization problem; 4) The maximum values of position and speed (inequality restrictions) are $0.2\ m$ and $0.2\ m/s$, respectively; 5) The minimum and maximum values for the time interval (h_i) between two successive points of the trajectory (lateral restrictions) are $0.0005\ s$ and $0.9\ s$, respectively; 6) The performance of the generated trajectory is evaluated by comparing the results obtained with the FMA and the 7th degree reference trajectory presented in [10]. The choice for this type of trajectory is because the cascade controller requires a trajectory of reference that is continuous, both for position, speed and acceleration. 7) For both paths, the same gains were used as those used in the controller by [10]; and, finally, 8) the path generated will henceforth be called TCH (Trajectory- Cylinder - Hydraulic).

To compare the trajectory of [10] and the TCH, the type of trajectory, the maximum flows in chambers A and B, the maximum acceleration and the maximum position obtained along the proposed route were analysed. Tab. I presents the results obtained in the 1st optimization stage, implementing both trajectories as a reference in the controller under study. The controller gains used in this analysis were determined by [10].

TABLE I
RESULTS OBTAINED IN THE 1ST OPTIMIZATION STEP.

	Trajectory by [10]	TCH
Type of trajectory	*7th order polynomial*	*5th-order B-spline*
Effective flow rate chamber A	$1.54 \times 10^{-5}\ m^3/s$	$1.35 \times 10^{-5}\ m^3/s$
Effective flow rate chamber B	$7.43 \times 10^{-6}\ m^3/s$	$6.53 \times 10^{-6}\ m^3/s$
Maximum acceleration	$0.11\ m/s^2$	$0.0673\ m/s^2$
Maximum position	$0.16\ m$	$0.16\ m$

The comparison between the trajectories interpolated with 7th order polynomials and with 5th order polynomials is valid because both cases guarantee the continuity of the position function and its three derivatives. This is a necessary requirement for the reference paths of cascade controllers [29].

Analyzing the results of Tab. I, it can be concluded that both trajectories meet the operational limits of the studied hydraulic actuator. Although the methods show similar behavior in terms of flow and position, it is important to note that the TCH generated maximum acceleration values significantly lower than the trajectory of [10]. It is understood that these results were obtained because the proposed optimization method considers the minimization of acceleration in the actuator's path (objective function). Fig.4 and Fig.5 show the position and acceleration curves obtained by applying both trajectories as a reference in the analyzed control algorithm. In this work, the pressure curves in the chambers are not presented.

The results obtained in this analysis show that both trajectories can be applied to the controller under study. TCH highlights the ease of implementation of the algorithm, regardless of the path of the actuator, and this trajectory also presents movements with less flow and acceleration.

In Fig.5, between 2 and 4 seconds, it is possible to observe that the trajectory proposed in [10] presents better results, because the understanding in this period is equal to zero. However, mean squared error and acceleration peaks are smaller in TCH during the operating cycle.

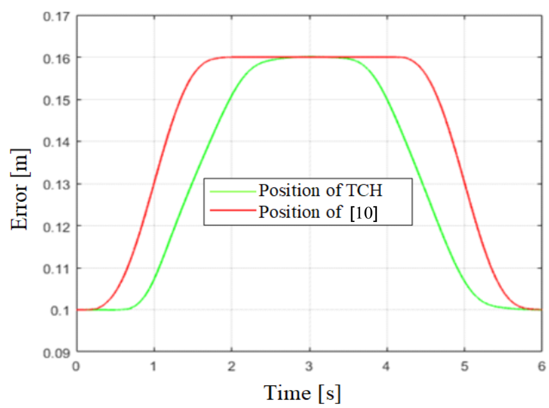

Fig. 4. Comparative graph of the trajectory tracking between TCH and [10] for the same controller.

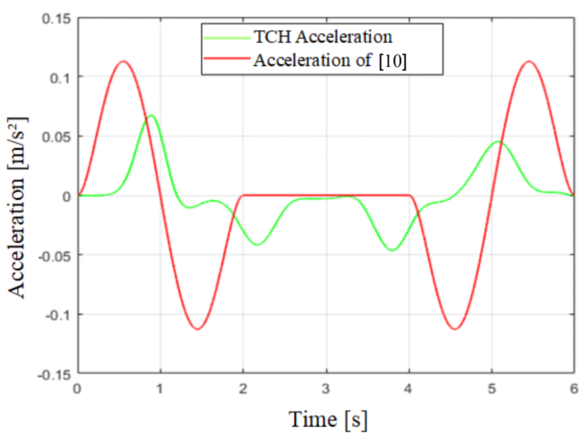

Fig. 5. Comparative graph of accelerations between TCH and [10] for the same controller.

Subsection V.B analyzes the implementation of the optimization technique for the cascade controller gains adjustment, comparing the trajectory tracking error obtained with the TCH with the optimized gains and the trajectory and gains of [10].

B. Gain Analysis for the Cascade Controller

In order to establish the ideal adjustment of the gains for the cascade controller, the present work proposes the use of an optimization algorithm based on the minimization of the actuator position error. Thus, for the trajectory tracking problem, presented in subsection IV.C, two situations are considered: the gains are empirically tuned [10] and the gains are obtained through the adopted strategy. Thus, the results of 2 simulations are presented. The first simulation consists of using the TCH and the empirical gains proposed by [10]. The second simulation uses the TCH as input to the controller, and the optimum gains of the control algorithm are obtained according to the procedure described in subsection IV.D. This trajectory is characterized by a useful distance of 0.06 m and with the completed cycle (advance and return of the actuator) of 6 seconds. The simulations for TCH applying the gains presented in [10] and the optimal gains are presented in Fig. 6.

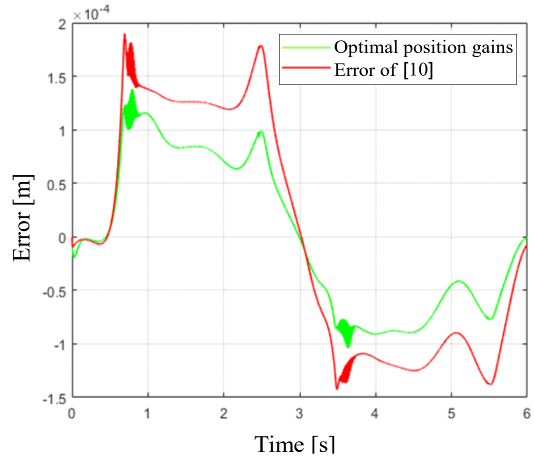

Fig. 6. Position errors using empirical tuning and optimized tuning, for a point-to-point trajectory.

It is possible to observe that in Fig.6 there is a small difference in the error of trajectory tracking comparing the errors obtained by the empirical method of adjustments of gains [10] and by the proposed optimization algorithm. This can also be seen in Fig.7, since the trajectories are practically overlapping, due to the low tracking error presented in both methods.

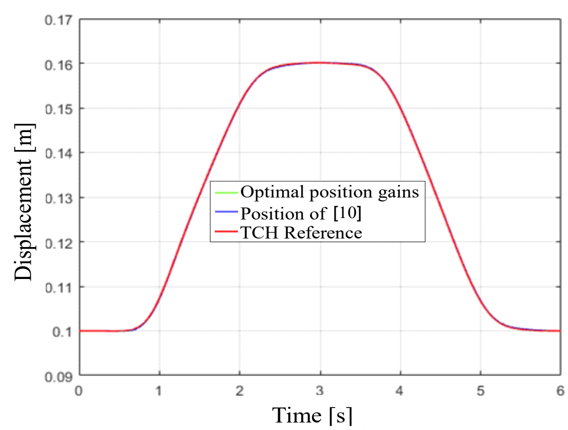

Fig. 7. Desired point-to-point trajectory and effector position.

An important aspect in the analysis of the efficiency of the reference parameters of a controller is to evaluate the control signal during the travel of the actuator, since high values of the control signal can saturate the actuators, influencing the tracking error. It is also important to mention that the control signal is proportional to the energy consumed by the system. Fig. 8 shows the valve control signal considering the gains shown in [10] and the optimum gains

The control signal for both methods showed a similar behavior. The quadratic mean (RMS) of both curves was 0.5131 [10] and 0.5134 for the proposed optimization method. These results indicate that the proposed technique for the optimization of gains meets the functionalities of the controller and hydraulic actuator under study. Although the control signals provide similar results, it is important to mention that the smallest tracking error occurs with the gain optimization technique.

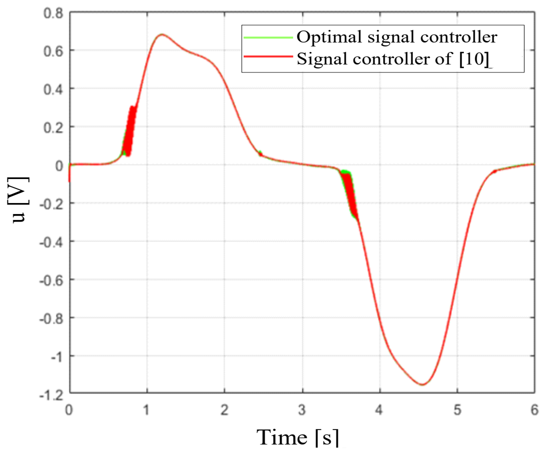

Fig. 8. Control signal for both gain adjustment methods.

The values of the gains applied in the cascade controller were obtained using TCH (reference). The maximum errors of trajectory tracking are shown in Tab. II. The maximum and minimum limits adopted for the controller gain values (lateral constrains) K_p, K_d and λ are [250 4100 350] and [15 500 30], where K_p, K_d and λ are the variables of the design vector. The optimal parameters provided by the FMA algorithm are: $n = 15, \alpha = 1, \beta_0 = 0.5, \gamma = 0.2$ and the maximum number of iterations is 10. The parameters for the FMA were determined through an empirical process of trial and error, observation, and experience. The stopping criteria for the FMA algorithm were established using three metrics: the coefficient of variation (CV) of the objective function for each individual, the Euclidean distance between the best solutions found in consecutive iterations, and the maximum number of iterations. These criteria were set to ensure that the algorithm converges to a solution in a reasonable amount of time. The convergence of the process can be measured by the CV, as if a global optimum is found, all individuals will be attracted to that solution, thus significantly reducing the CV.

TABLE II
GAINS OBTAINED IN THE 2ND OPTIMIZATION STEP.

	Empirical gain values [10]	Optimized value gains
K_p	200	46.01
K_d	5000	3891.38
λ	150	335.96
Maximum trajectory tracking error	$1.8998 \times 10^{-4}\ m$	$1.3806 \times 10^{-4}\ m$

VI. DISCUSSION AND CONCLUSIONS

The results of the simulations show that the proposed method is suitable for generating optimal reference trajectories and for tuning the gains of cascade control algorithms. It is also observed that the proposed method can be applied to different types of control strategies and servo-positioning systems, since it is necessary to supply to the optimization algorithm only the points to be covered by the actuator and its physical limits, considering the 1st optimization step.

For the 2nd stage of optimization, it is necessary to provide only the maximum and minimum values of the gains that the controllers support. It is important to mention that the result regarding the implementation of the two optimization steps, results in a set of points that are used with reference to the controller of the hydraulic servo-positioner. The implementation of optimization techniques considering the specific plant and control system, show that TCH presents a behavior similar to the trajectory presented in the work of [10]. However, the TCH provides lower values of acceleration and flow when applied to the controller under study. This result does not allow to infer which trajectory is most suitable for this system, since there were no significant differences in the interpolation of the analyzed intermediate points. Still, the analyzed trajectories are not overlapping. This is due to the fact that the trajectory of [10] is of the point-to-point type, interpolated in a time interval of 6 seconds, not considering any type of time parameter for the interpolation of the points.

The results analysis in subsection V.B shows that the proposed iterative technique for tuning the controller gains resulted in a 27% reduction in the trajectory error compared to the gain adjustment method presented in [10]. Despite the significant difference in the errors of tracking the trajectory provided by the analyzed methods, there were no major variations in the control signal of the valve, which consists of an indicator of the amount of energy consumed by the system. The comparative analysis of the gains was carried out using the TCH as a reference signal for the cascade controller. The implementation of the optimization algorithms proposed in steps 1 and 2 requires high computational cost of simulation.

The optimal adjustment of the gains in controllers applied to hydraulic servo actuators has proven to be a crucial aspect in ensuring the performance and stability of these systems. Future work could focus on exploring advanced optimization algorithms, incorporating more complex models of the hydraulic system, and considering multiple objectives to achieve an optimal trade-off between performance and robustness. Additionally, investigating the impact of different control strategies, such as robust control and model predictive control, on the optimization of gain parameters could provide valuable insights into the design of high-performing hydraulic servo actuator systems.

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