Towards Unique Circuit Synthesis of Power Transformer Winding using Gradient and Population based Methods

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***Abstract*—The aim of the paper is to synthesize a nearly unique, physically realizable and mutually coupled ladder circuit representation of a two-winding transformer by identifying the most accurate and reliable optimization technique such that prior knowledge in selecting the initial guess and search space is avoided. To this end, magnitude and phase of the driving-point impedance function are captured by performing sweep frequency response analysis (SFRA). Then, two widely used population-based algorithms namely, particle swarm optimization (PSO) and artificial bee colony (ABC) and one gradient-based sequential quadratic programming (SQP) algorithm are implemented. Two case studies are considered namely, i) 15 MVA, 66/11.55 kV transformer and, ii) 111 kVA, 7.33/1.22 kV transformer. The performance of the aforementioned algorithms is compared using three evaluation parameters namely, repeatability, accuracy and reliability.**

***Index Terms*—Circuit synthesis, frequency response analysis, optimization, power transformer.**

I. Introduction

The breakdown of transformers due to abnormal voltages and short-circuit forces is a serious concern in the power system. Apart from this, transportation and rough handling of transformers are also responsible for reducing mechanical strength of the transformer. Therefore, the condition assessment of the transformer is essential [1]–[4]. Frequency response analysis (FRA) is a technique that has grasped popularity in monitoring the mechanical integrity of the transformer winding [5]. To increase the quantitative interpretation of the condition of transformer winding using FRA data, numerical indices and artificial intelligence are utilised [6]–[8]. As the mechanical deformation in the winding gets reflected as changes in the high-frequency behaviour of the transformer, characterizing its high-frequency behaviour is essential. For this purpose, physically realizable ladder circuit corresponding to the high-frequency behaviour of the transformer winding can be built from the FRA data [9]–[11]. However, synthesizing the ladder circuit model is challenging and is not straightforward since all circuit variables cannot be estimated directly from the terminal data.

In order to estimate such circuit variables accurately and efficiently, several optimization algorithms have been implemented. The Genetic Algorithm (GA) is utilised for identifying circuit variables of the transformer [10]–[12]. In [13], modelling of the power transformer winding is achieved using GA and Bacterial Swarming Algorithm (BSA). The Particle Swarm Optimisation (PSO) algorithm is implemented for the circuit variables identification [14]–[16]. In [17], the PSO algorithm is used for the estimation of transformer winding geometry by utilising the FRA data. It is reported in [15]–[17] that the PSO algorithm is simple and precise. Moreover, PSO appeared to be the best in identifying circuit variables. In [18], Artificial Bee Colony (ABC) is introduced to identify variables of the disc winding. It is concluded that the ABC algorithm is capable of obtaining nearly unique and physically realizable ladder network such that there exists a good match between measured and estimated natural frequencies. From the above literature, it can be observed that the population-based algorithms are gaining importance due to the application of social behaviour on real-world problems. In contrary, gradient-based Sequential Quadratic Programming (SQP) algorithm for synthesizing the ladder circuit is presented in [19], [20]. From the literature, following observations can be made:

Selection of best-estimated solution: Several optimization algorithms including population-based and gradient-based have been proposed in the last two decades to synthesize the ladder circuit. However, the inspection of repeatability, accuracy and reliability aspects of the circuit variables estimated by algorithms have been ignored. Further, the mentioned three aspects will be essential in selecting the best-estimated solution since it is computed in an iterative manner. With this in mind, the authors propose comparative study on the performance analysis of the two widely accepted population-based (PSO, ABC) algorithms with that of the gradient-based (SQP) algorithm considering the three mentioned aspects.

Implementation on actual two-winding transformer: The PSO and ABC algorithms discussed in the literature are implemented only on single and isolated windings. However, in practice, the actual, single-phase transformers have many windings and hence, addressing the multiple ladder circuit model is essential. Thus, in the proposed work, the applicability and performance aspects of the considered algorithms are examined with regard to two-winding transformer.

Foreknowledge of algorithm parameters: In [15], [16], [18], it is observed that the parameters of the population-based algorithms such as population size, inertia weight, acceleration constants, etc. are chosen based on the experience or, trial and error method. Further, human intervention is necessary in order to finalise the search space and the initial guess. Whereas, in [19], [20], the details regarding the parameter adjustment for the gradient-based algorithm are not discussed. Thus, in this work, an attempt is made to compare the performance of the

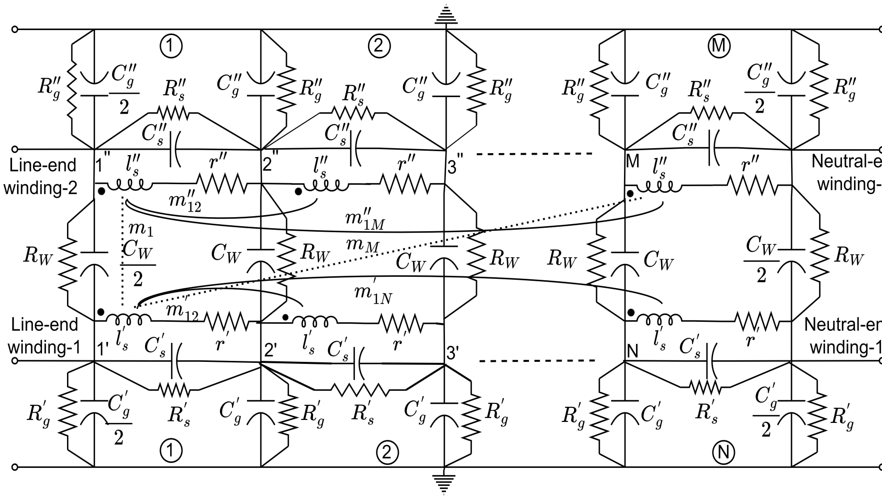


Fig. 1. Equivalent ladder circuit of a uniform two-winding transformer, where, $l_s^{'}$, $l_s^{''}$, $C_s^{'}$, $C_s^{''}$, $C_g^{'}$, $C_g^{''}$ are sectional values of self-inductances, series-capacitances and shunt-capacitances of winding-1 and winding-2 respectively.

population-based and gradient-based algorithms with regard to parameter adjustments. Therefore, the objectives of the proposed work are, i) selection of the best-estimated solution, ii) synthesis of a nearly unique circuit model that represents high-frequency behaviour of the two-winding transformer and, iii) discussion on the parameters adjustment criteria of the considered algorithms.

II. Formulation of an optimisation problem

In order to estimate the circuit parameters (variables), an optimisation problem can be framed. Let f be considered as the function of 'n' number of variables with the variable vector symbolized as $\overline{\boldsymbol{x}} = (x_1, x_2, \ldots, x_n)^T$. It is required to maximise/minimize the objective function ($f(x)$) within the limits defined over variables of $\overline{\boldsymbol{x}}$. The defined limits of variables considered are usually referred as inequality constraints. Sometimes, additional governing equations with equality sign are also considered for meeting certain criteria that are usually referred as equality constraints. The problem of finding an appropriate solution for all the considered variables to maximise/minimize $f(x)$ satisfying all the constraints is referred as an optimisation problem. An initial guess is required to initiate the search algorithm. Thus, the performance of any optimisation algorithm depends on how objective function, design variables, number of sections, constraints, initial bounds are selected. Hence, these parameters are to be decided judiciously as explained below.

Number of variables: The circuit topology for a uniform two-winding transformer can be represented as shown in Fig. 1. The number of sections corresponding to winding-1 and winding-2 are N and M respectively. These can be estimated in an iterative manner with lower limit as the number of visible peaks p in the driving-point impedance (DPI) plot [19].

It can be found in [20] that the complex ladder circuit (corresponding to multiple windings with inter-winding capacitances, C_w and mutual inductances, m) could be reduced to a single ladder circuit as shown in Fig. 2 if the neighbouring windings are shorted and grounded. With such arrangements,

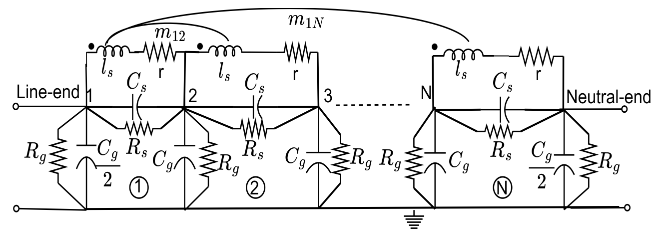


Fig. 2. Reduced equivalent ladder circuit of a two-winding transformer.

the shunt capacitance pertaining to the reduced ladder network (C_g) would become the combination of shunt capacitance of the winding under test (either $C_g^{'}$ or $C_g^{''}$) and C_w. Thus, the variable corresponding to the ladder circuit model as shown in Fig. 2, can be defined as, $\overline{\boldsymbol{x}} = (l_s, m_{1j}, \forall j = 2, \ldots, N, C_g, C_s)^T$, $x \in R^n$. Therefore, the dimension of the problem (n) is equal to number of variables to be estimated and is, $N+2$.

Initial bounds: These are helpful to limit the search space. Since the synthesized circuit should be physically realizable, all the variables should be positive values. Thus, the lower bound (lb) can be considered as zero (or close to zero). The upper bound (ub) can be any realistic value greater than zero.

Number of sections: In [16], the number of sections are chosen same as the number of visible peaks p in the DPI plot. However, in [21], it is reported that the some peaks might not be visible in the DPI plot due to pole-zero cancellation even though they exist. Considering this, N is estimated in an iterative manner with lower limit as p.

Objective function: Objective function plays a vital role in the convergence. In many literature, the difference between the measured FRA magnitude (acquired from transformer terminals) and estimated FRA magnitude (obtained from the ladder circuit) is considered [13], [15]. Whereas, some literature discuss about difference between the measured and estimated natural frequencies [18]. The natural frequencies depend only on capacitances and inductances (influence of resistance is minimum). Such decoupling of resistance helps in reducing the computational complexity and simplifies the parameter estimation. To this end, the later approach is considered to frame an objective function as shown below.

$$\sqrt{\frac{\sum_{i=1}^{p}(ocnf_i^m - ocnf_i^e)^2}{p}} + \sqrt{\frac{\sum_{i=1}^{p-1}(scnf_i^m - scnf_i^e)^2}{p-1}} \quad (1)$$

where, ocnf^e, scnf^e are the estimated open-circuit and short-circuit natural frequencies corresponding to ladder circuit model and ocnf^m, scnf^m are the measured open-circuit and short-circuit natural frequencies corresponding to actual transformer winding.

Constraints: The information on equivalent capacitance and equivalent shunt capacitance is beneficial to estimate C_g and C_s. However, acquiring these parameters from the terminal measurement is challenging as they are strongly coupled with the inductances. Therefore, these parameters are to be carefully estimated from the terminal data [19], [20]. Let equivalent capacitance and equivalent shunt capacitance obtained from the terminal data be denoted as EC^m and ESC^m respectively and that estimated from the circuit be denoted as EC^e, ESC^e respectively. To improve the search process, the constraints (f_{EC}(x)) can be formulated as percentage error between EC^m and EC^e, and (f_{ESC}(x)) as percentage error between ESC^m and ESC^e.

There exists mutual inductance between any two coils of the winding as well as self inductance of the coil as shown in Fig. 2. In addition to this, the mutual inductance between the coils decreases when the distance between them increases. To satisfy this condition, the constraints are formulated as,

$$m_{1i} > m_{1j}, \forall i, j = 1, \ldots, N,\ j > i \quad (2)$$

$$al_s < m_{12} < bl_s \quad (3)$$

$$am_{1i} < m_{1(i+1)} < bm_{1i}, \forall i = 2, \ldots, N. \quad (4)$$

Thus, by establishing appropriate correlation between self- and mutual-inductances, the search space can be confined and estimation can be improved. Moreover, the information on equivalent inductance (EL) is beneficial to judge the correctness of the estimated self- and mutual-inductance. However, estimation of EL from the terminal data is not straightforward and is to be carefully estimated [22]. Let the equivalent inductance obtained from the terminal data be denoted as EL^m and that estimated from the circuit be denoted as EL^e respectively. With this, the f_{EL}(x) can be formed as the percentage error between EL^m and EL^e. The limits a and b can be adjusted iteratively such that f_{EL}(x) is minimum. The considered objective function depends on the natural frequencies and hence, it will not be influenced by the resistances as discussed earlier. However, to obtain the damping associated with natural frequencies, resistance (in series with inductance) of each section is approximated as, (r) = $\frac{\text{measured dc resistance } (R_{dc})}{N}$. Resistance in parallel with capacitor (R_s and R_g) can be of any highest value, here, it is approximated to 100 kΩ.

III. Optimisation algorithms

In the work, two population-based PSO and ABC algorithms and one gradient-based SQP algorithm are utilised to construct a ladder network. The brief description of each algorithm is given as follows.

A. Population-Based Algorithms

1) Particle swarm optimization: It is initially developed by Kennedy and Eberhart based on the inherent behaviour of bird flocking [16]. The criteria for searching optimal values will be according to the natural phenomena of swarm intelligence in which position and velocities of particles are updated in an iterative manner [23]. Several advancements over standard PSO algorithm has been reported to meet the requirement of early convergence by placing inertia weights, acceleration factors and other appropriate methodologies [24]. It can be found in [16] that the particle swarm optimisation has capability to find the global solution with a fast convergence rate. The generalised steps of implementing the PSO algorithm can be found in [16].

2) Artificial bee colony: It is an optimization algorithm based on the intelligent foraging behaviour of honey bee swarm, proposed by Karaboga in 2005 [25], [26]. The algorithm undergoes into three phases at each iteration to deliver the best food source or solution. The same algorithm is employed in [18] to synthesize the equivalent parameters of ladder circuit with a population size of 16. The algorithmic steps for synthesizing circuit parameters can be found in [18].

B. Gradient based Sequential Quadratic Programming

Gradient search methods are gaining importance over existing population-based methods as the algorithms are robust and independent of few assumptions that are usually considered in the case of population-based methods. The prerequisites and the mathematical background associated with gradient search method are discussed in the subsequent part of the article. It can be seen that the objective function and constraints exhibit non-linear relationship with the variables. Hence, it is necessary to implement gradient-based algorithm that make use of non-linear programming technique. A non-linear programming problem is thus defined by considering the previously discussed objective function and constraints.

Selection of the algorithm: It is evident (shown in [27]) that the SQP algorithm has achieved huge success in solving constrained non-linear optimization problem. The algorithm involves Quadratic Programming (QP) to solve the non-linear optimization problem in an iterative manner. The module pertaining to QP is explained as follows:

$$\begin{aligned} &\underset{d \in R^n}{\text{minimize}}\ \nabla f(x_k)^T d + \frac{1}{2} d^T H_k d \\ &s.t. \\ &h(x_k) + \nabla h(x_k)^T d = 0 \\ &g(x_k) + \nabla g(x_k)^T d \leq 0 \end{aligned} \quad (5)$$

Here, d is $x - x_k$ and $\nabla f(x)$ (vector of partial derivative) is the gradient of f at $x \in R^n$. This is basically n-dimensional vector as shown below.

$$\nabla f(x) : \left(\frac{\partial f(x)}{\partial x_1}, \ldots, \frac{\partial f(x)}{\partial x_n} \right)^T \tag{6}$$

The Hessian matrix of f at $x \in R^n$ can be represented by,

$$\mathbf{H}(\mathbf{x}) = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial^2 x_1} & \cdots & \frac{\partial^2 f(x)}{\partial x_1 \partial x_n} \\ \vdots & & \vdots \\ \frac{\partial^2 f(x)}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f(x)}{\partial^2 x_n} \end{bmatrix}$$

It is a matrix comprising of second partial derivatives of an n-variable objective function. The SQP algorithm makes use of a quasi-Newton approach. At each iteration, the gradient is computed using finite differences and Hessian is approximated using appropriate update criteria. This information is utilised to generate a QP sub problem as indicated above.

Selection of the solver: After identifying an algorithm, the subsequent task is to choose the solver. It can be seen from the decision table in MATLAB that the solver '$fmincon$' can be utilised for any smooth, non-linear objective function with constraints. In this work, $fmincon$ solver and SQP algorithm are utilised to solve QP sub-problem. The solution produces a vector d_k which is used further to update the parameter in an iterative manner.

$$x_{(k+1)} = x_k + \alpha_k d_k \tag{7}$$

Here, α_k is the step length parameter. It is determined by line search procedure so as to fulfill the minimization criteria of the defined objective function. Ultimately, algorithm finds an optimum solution of the defined problem such that the Karush-Kuhn-Tucker (KKT) conditions are satisfied and Hessian is positive definite.

In each iteration, gradient is computed and an approximation is made of the Hessian (of the Lagrangian function) using a quasi-Newton updating method. Then, the QP sub problem is solved. At the end, it utilises the solution to build new iterate $x_{(k+1)}$. This process is carried out such that the sequence x_k converges to a local minimum (x^*).

The program pertaining to PSO, ABC and SQP algorithms are written in MATLAB. The PSO and SQP algorithms are developed using $particleswarm$ and $fmincon$ commands [28], [29]. Whereas, the MATLAB files of the ABC algorithm are taken from [30]. Further, as per the requirements, the source codes are modified by adding objective and constraint functions, bounds, etc.

IV. Selection criteria

In order to select the best optimization algorithm, three evaluation parameters namely repeatability, accuracy and reliability are considered as described below.

Repeatability: To understand the repeatability aspect, the algorithms can be executed several times and best five executions in terms of lowest f(x) value can be considered. Later, ladder circuit variables corresponding to five selected executions can be recorded. Further, the recorded variables can be plotted using the box plot. According to the box plot, if more is the thickness of the box, then more is the deviation in the estimated circuit variables. In contrary, lesser the thickness of the box then lesser is the deviation.

Accuracy: To check the accuracy of the algorithm, the % error is calculated between measured terminal quantities and the mean of terminal quantities (EL, EC and ESC) estimated in five executions. If the % error of each quantity is found nearly zero, then the corresponding algorithm is considered to be accurate.

Reliability: The algorithm can be considered reliable if the estimated circuit variables are both repeatable and accurate.

Thus, a careful comparison with regard to repeatability, accuracy and reliability utters about the best performing algorithm. The flowchart associated with synthesizing the equivalent circuit of transformer winding using PSO, ABC and SQP algorithms is shown in the Fig. 3.

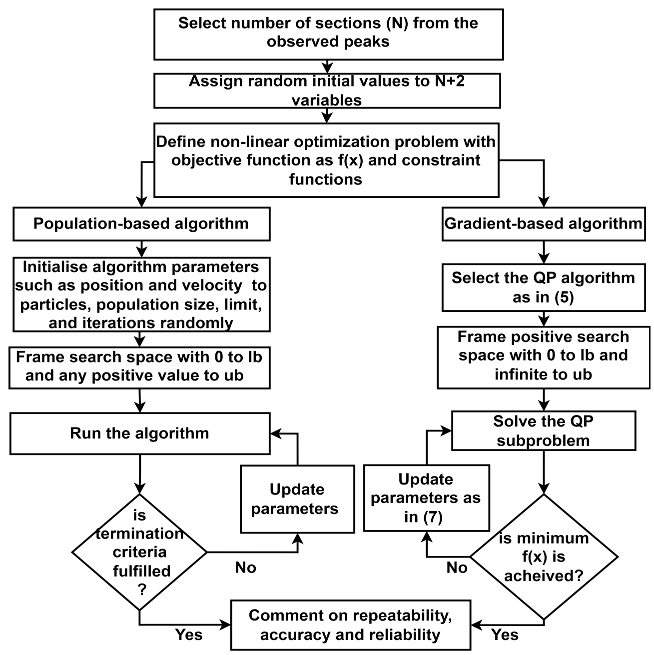


Fig. 3. Flowchart of considered algorithms.

V. Experimental Investigation

The research work focuses on the correct estimation of the winding parameters from the terminal data. To validate the effectiveness of the algorithms, a 15 MVA and 111 kVA, single-phase, two-winding transformers are considered. The instruments used to obtain the frequency response of the winding are i) a 0-20 V_{pp}, 10 Hz-25 MHz function generator to provide the input excitation, ii) an 8-bit, 2 GSamples/s digital oscilloscope to record responses, and iii) a 2 mV/mA current probe with bandwidth 450 Hz-60 MHz to measure input current. All signals are measured using 50 Ω coaxial cables. The algorithms are executed in MATLAB environment supported by INTEL(R) CORE(TM) i5-3340M CPU at 2.70 GHz, 8-GB RAM.

A. 15 MVA, 66/11.55 kV, Two-Winding Transformer

The considered winding is a disc type HV and disc type LV windings of a 15 MVA, 66/11.55 kV, three-phase power transformer as shown in Fig. 4. The interconnected taps are braced to get the entire HV winding for the measurement. To reduce the influence of non-tested LV winding, its terminals are shorted and grounded to the core. With the neutral terminal of the HV winding under test is grounded to the core, the FRA is performed. Corresponding magnitude plot is shown in Fig. 5 with the solid line.

Fig. 4. 15 MVA two-winding transformer.

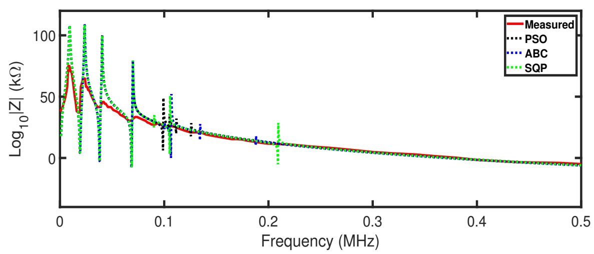


Fig. 5. DPI magnitude of 15 MVA transformer (solid line) and synthesized circuits (dotted lines).

TABLE I
NATURAL FREQUENCIES (MHZ) OF THE 15 MVA TRANSFORMER

ocnf^m	0.009	0.023	0.041	0.072
scnf^m	0.019	0.038	0.068	

TABLE II
ALGORITHM PARAMETERS: 15 MVA TRANSFORMER

Parameter	PSO	ABC	SQP
W	[0.35,0.9]	-	-
limit	50	242	-
C_1	0.6	-	-
C_2	0.9	-	-
pop. size	30	44	-
max.iter	1000	1500	50
lb, ub	1e^{-4}, 12	1e^{-4}, 12	0, inf
a, b	0.1, 0.8	0.1, 0.8	0.1, 0.8

The acquired ocnf^m and scnf^m are listed in Table I. Further, R_{dc}, EL^m, ESC^m and EC^m are found as, 10 Ω, 382 mH, 1.7 nF, and 0.43 nF, respectively. As the observable peaks are 6, N can be chosen $\geq$ 6. The final value of N is found as 9. Thus, there exists 11 variables namely, l_s, m_{1j} $\forall$ j=2,...,N,

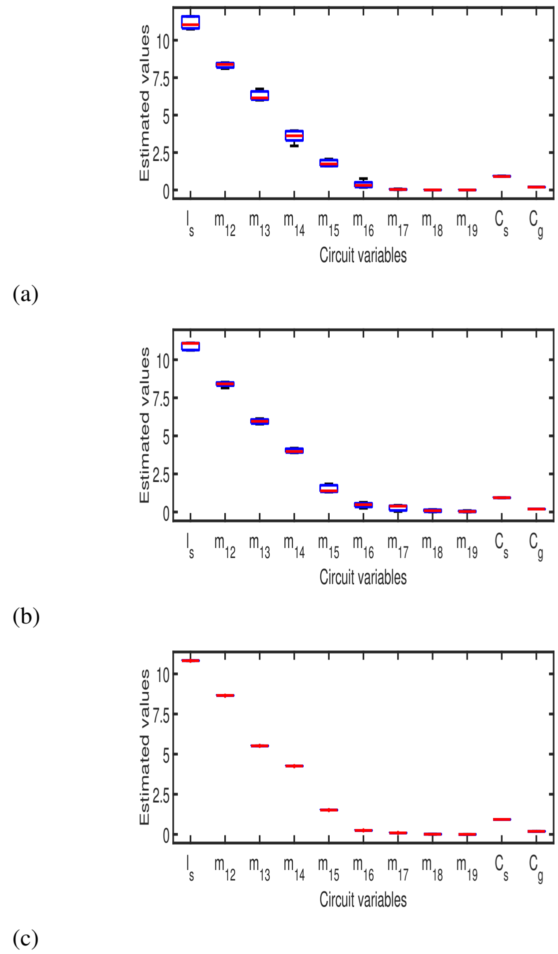


Fig. 6. Box plot representing repeatability of self- and mutual-inductances (mH) and capacitances (nF) of 15 MVA transformer estimated by (a) PSO, (b) ABC, and (c) SQP algorithms.

C_s and C_g. The defined circuit variable are estimated by passing natural frequencies and EL^m, EC^m, ESC^m to PSO, ABC and SQP algorithms. To achieve optimum solutions, parameters pertaining to PSO algorithm such as population size (pop.size), inertia weight (W), acceleration coefficients (C_1 and C_2), maximum iterations (Max.Iter) and bounds (lb and ub), parameters belonging to ABC algorithm such as pop.size, Max.Iter and bounds are adjusted in an iterative manner. Whereas, for the SQP algorithm, only Max.Iter is adjusted with lb as zero and ub as infinite. Further, the common parameters to all algorithms are, limits (a,b) which are adjusted only once in an iterative manner and same values can be utilised in other algorithms. Moreover, in the population-based algorithms, bounds are kept same whereas, maximum iterations and population size are adjusted independently to acquire the optimized results. With the finalised parameters as shown in Table II, the algorithms are executed several times and best five executions are recorded. To check the repeatability of estimated variables, the recorded variables are plotted using the box plot as shown in Fig. 6.

From Fig 6, it is clear that the estimated circuit variables corresponding to PSO and ABC algorithms are deviated in

TABLE III
MEAN OF ESTIMATED CIRCUIT VARIABLES (mH, nF) AND EXECUTION TIME (s): 15 MVA TRANSFORMER

Algorithm	l_s	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{18}	m_{19}	C_s	C_g	t_e
PSO	11.15	8.33	6.2925	3.5737	1.7938	0.3763	0.0384	0.0041	0.0011	0.9132	0.1908	43
ABC	10.9046	8.3879	5.9480	4.0232	1.5199	0.4548	0.2754	0.0816	0.0374	0.9362	0.1908	60
SQP	10.8229	8.6550	5.5163	4.2572	1.5140	0.2458	0.0917	0.0094	0.0009	0.9304	0.1889	5

TABLE IV
MEAN AND % ERROR OF ESTIMATED NATURAL FREQUENCIES (MHz) AND TERMINAL QUANTITIES (mH, nF): 15 MVA TRANSFORMER

Quantity	PSO		ABC		SQP	
	Mean	% error	Mean	% error	Mean	% error
$ocnf_1^e$	0.0092	2.2	0.0092	2.2	0.0093	3.3
$ocnf_2^e$	0.0236	2.609	0.0236	2.609	0.0236	2.609
$ocnf_3^e$	0.0405	1.21	0.0404	1.46	0.0404	1.46
$ocnf_4^e$	0.0697	3.19	0.0697	3.19	0.0701	2.64
$scnf_1^e$	0.0192	1.053	0.0193	1.579	0.0193	1.579
$scnf_2^e$	0.038	0	0.038	0	0.038	0
$scnf_3^e$	0.0684	0.588	0.0685	0.735	0.0689	1.324
EL^e	385.8	1.0	384.8	0.733	382	0
EC^e	0.43	0	0.43	0	0.43	0
ESC^e	1.7172	1.01	1.7172	1.01	1.7	0

five executions. In contrast, it is found that the deviation is negligible for the variables estimated by the SQP algorithm. The mean of the recorded circuit variables and execution time (t_e) are listed in Table III. Further, the approximated values of r, R_s and R_g are 1.11 Ω, 100 kΩ and 100 kΩ respectively. To comment on the accuracy of considered algorithms, the % error has been calculated. Corresponding values are tabulated in Table IV. From the results, it is noticed that the % error corresponding to the EL estimated by PSO, ABC and SQP is 1%, 0.73%, and 0% respectively. In the case of the EC, the % error is found as 0% for three algorithms. Whereas, in case of ESC, the % error values are 1.012%, 1.012% and 0% respectively. From these values, it is evident that among population-based algorithms, the ABC algorithm is found to be accurate. However, from the same values, the SQP algorithm is found to be more accurate than the ABC algorithm. Thus, the SQP algorithm is considered to be reliable. Further, the DPI plots of synthesized ladder networks with PSO, ABC and SQP algorithms are shown in Fig. 5 with dotted lines.

B. 111 kVA, 7.33/1.22 kV, Two-Winding Transformer

To understand the influence of neighbour windings, core, and tank on the results, a 111 kVA, 7.33/1.22 kV, dry type, single-phase, two-winding transformer with iron-core and tank is considered. The type of HV and LV windings are crossover and helical respectively. The LV winding is wound on the ferromagnetic core and the outer terminals of the HV winding are available for accessibility. The winding-core structure is covered by the aluminum tank without oil. There are 7 layers in the HV winding and each layer has 5 coils. The height, inner diameter, and outer diameter of the HV winding are 545 mm, 255 mm, and 307 mm respectively. The LV winding has 2 layers. The height, inner diameter, and outer diameter of the LV winding are 595 mm, 183 mm, and 217 mm respectively. To capture the terminal response of the HV winding, FRA is performed from few Hz to 1 MHz. While doing, the LV winding is shorted and grounded to reduce its influence on the measured FRA. Further, the transformer core, tank, LV winding and neutral terminal of the HV winding are connected at one point to emulate the ground plane.

The acquired magnitude plot of DPI as shown in the Fig. 7

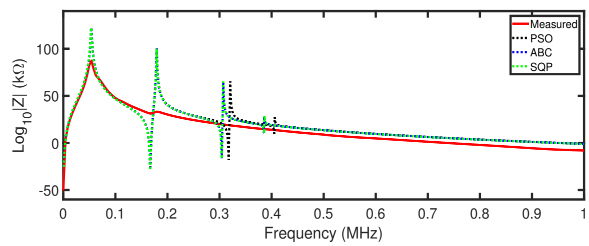
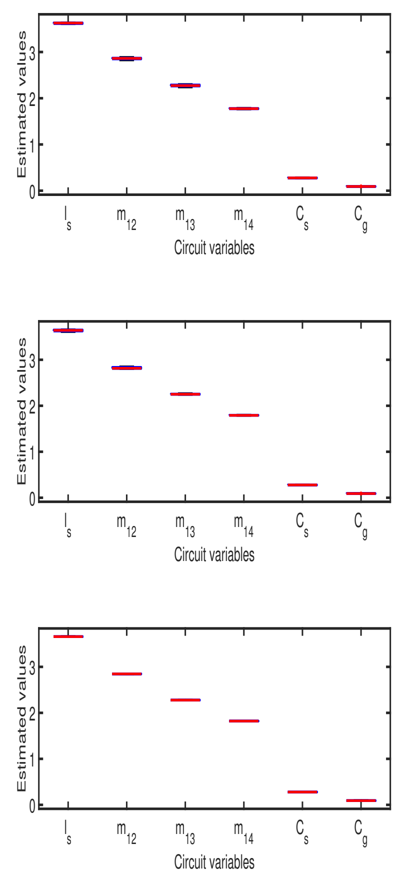


Fig. 7. DPI magnitude of 111 kVA transformer (solid line) and synthesized circuits (dotted lines).



(a)

(b)

(c)

Fig. 8. Box plot representing repeatability of self- and mutual-inductances (mH) and capacitances (nF) of 111 kVA transformer estimated by (a) PSO, (b) ABC, and (c) SQP algorithms.

TABLE V
ALGORITHM PARAMETERS: 111 KVA TRANSFORMER

Parameter	PSO	ABC	SQP
W	[0.4 0.9]	-	-
limit	50	60	-
C_1	0.65	-	-
C_2	0.9	-	-
pop.size	20	20	-
Max.Iter	1000	2000	50
lb, ub	$1e^{-4}$, 4	$1e^{-4}$, 4	0, inf
a, b	0.4, 0.8	0.4, 0.8	0.4, 0.8

TABLE VI
MEAN OF ESTIMATED CIRCUIT VARIABLES (MH, NF) AND EXECUTION TIME (S): 111 KVA TRANSFORMER

Algorithm	l_s	m_{12}	m_{13}	m_{14}	C_s	C_g	t_e
PSO	3.623	2.859	2.274	1.775	0.276	0.0911	25
ABC	3.636	2.8224	2.2517	1.7926	0.278	0.0909	35
SQP	3.657	2.8466	2.2773	1.8218	0.2787	0.0917	7

TABLE VII
MEAN AND % ERROR OF ESTIMATED NATURAL FREQUENCIES (MHZ) AND TERMINAL QUANTITIES (MH, NF): 111 KVA TRANSFORMER

Quantity	PSO		ABC		SQP	
	Mean	% error	Mean	% error	Mean	% error
$ocnf_1^e$	0.0537	0.55	0.0538	0.37	0.0534	1.1
$ocnf_2^e$	0.179	0	0.179	0	0.179	0
$scnf_1^e$	0.167	0	0.167	0	0.167	0
EL^e	44.291	0.38	44.071	0.875	44.460	0
EC^e	0.169	0.59	0.169	0.59	0.170	0
ESC^e	0.364	0.82	0.364	0.82	0.367	0

with the solid line. Two $ocnf^m$ are found as 54 kHz and 179 kHz and one $scnf^m$ is found as 167 kHz. The values of R_{dc}, EL^m, ESC^m and EC^m are found as, 4 Ω, 44.46 mH, 0.367 nF, and 0.17 nF, respectively. As the observable peaks are 2, N can be chosen ≥ 2 and the final value of N is found to be 4. For 4-section circuit, there exists 6 variables namely, l_s, m_{1j} $\forall$ j=2,. . . ,N, C_s and C_g. The values of the adjusted parameters of the algorithms are shown in the Table V. The mean of the estimated circuit variables and execution time for five successful executions are tabulated in the Table VI. Moreover, the approximated values of r, R_s and R_g are 1Ω, 100 kΩ and 100 kΩ respectively. The box plot shown in Fig. 8 represents the repeatability aspect. From Fig. 8, the deviation in estimated variables by all three algorithms is found to be insignificant. Therefore, all three algorithms are considered to be repeatable.

To address the accuracy aspect, the % error has been calculated and tabulated in Table VII. In all the three algorithms, mixed results have been observed. Among population-based algorithms, the PSO estimated EL and ESC error is found lesser compared to the corresponding values estimated by the ABC algorithm. Whereas, in the case of EC, the ABC algorithm is found to be accurate. In the case of the SQP algorithm, the % error regarding EL, EC and ESC is found to be 0% as shown in Table VII. As a result, the SQP algorithm is considered to be reliable. Further, the ladder network is synthesized from the mean of variables estimated by PSO, ABC and SQP algorithms. Corresponding DPI plots are shown in Fig. 7 with dotted lines.

From the above two case studies, it is evident that the algorithms are capable of estimating the circuit variables, however, mapping the amplitudes of measured and estimated DPI plots cannot be achieved because of the following reasons.

- The winding resistance (represented in series with an inductor) is considered to be ($\frac{R_{dc}}{N}$). However, its variation with respect to frequency is ignored in this work.
- The resistors across C_s and C_g i.e., R_s and R_g, respectively are approximated to 100 kΩ since these cannot be acquired directly from the terminals.

The task of accurate mapping of amplitudes of measured and estimated DPI plots; will be performed separately and it will be a part of the future work.

VI. FUTURE SCOPE

The work was demonstrated on synthesizing nearly a unique ladder circuit model of a transformer windings under healthy conditions. Therefore, in the future, the work can be extended i) to synthesize the ladder network of winding under faulty conditions and, ii) to map both amplitude and natural frequencies of the measured and estimated DPI plots.

VII. CONCLUSIONS

In this work, a comparison is performed between population-based and gradient-based algorithms to construct an accurate and nearly unique high-frequency ladder circuit of the two-winding transformer from the frequency response measurements. The capability of algorithms are assessed in mapping the actual and estimated ladder network variables using three aspects such as repeatability, accuracy and reliability. The performance of three considered algorithms is demonstrated by conducting the FRA on two windings namely, 15 MVA, 66/11.55 kV transformer and 111 kVA, 7.33/1.22 kV transformer respectively. From the comparative study, it is identified that the circuit variables estimated by PSO, ABC, and SQP algorithms are found repeatable in 111 kVA transformer. In 15 MVA transformer, the circuit variables estimated by PSO and ABC algorithms have deviated. As per the accuracy aspect, the SQP algorithm outperforms the PSO and ABC algorithms. Hence, it is considered as the reliable algorithm. Secondly, although three algorithms are certainly capable of estimating circuit variables, PSO and ABC algorithms require either human involvement with domain knowledge or the trial and error approach to define population size, inertia weights, bounds, etc. In contrast, this effort is reduced to a greater extent in the SQP algorithm. Further, the execution time of the SQP algorithm is identified lesser than the population-based algorithms. For these reasons, the SQP algorithm can be considered appropriate and efficient.

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