# Stochastic Modeling and Analysis of the Energy Consumption of Wireless Sensor Networks

Felipe P. Correia, Marcelo S. Alencar and Karcius D. R. de Assis

***Abstract*—Energy management in Wireless Sensor Networks (WSN) is a challenging problem that calls for careful modeling and analysis. It is shown in the paper that the problem can be more precisely characterized by calculating the probability distribution function (PDF) and the maximum and minimum values of the energy consumed by the network. As a result, this paper provides a novel approach for modeling and assessing the probability range of energy expended by WSN nodes. The steps that were taken for developing this project were: (1) an initial investigation into the power consumption of WSN devices; (2) the proposal of a stochastic model for consumption; (3) the collection of data using the LoRaWANSim simulator; and (4) the interpretation and comparison of simulation with theoretical results. The examination of the suggested method with a discrete-event simulator and the resulting mathematical expressions provide a deeper insight into the energy consumption patterns of WSNs.**

***Index Terms*—Internet of things, wireless sensor networks, energy consumption, stochastic model.**

## I. Introduction

A WSN is an infrastructure formed by equipment for measuring, controlling, and communicating with a central station. The user can react to events related to the monitored phenomenon based on the information received. The WSNs have low consumption at the wireless nodes, with limited memory and processing capacity, and include communication systems, interface nodes, and application servers. The hardware that makes up a WSN device is called a node, sensor node, mote, or end-device (ED) [1], [2]. The server software handles data and events, generates reports, and can interact with the network, sending commands to the network actuators [3].

Energy saving is still a challenge for WSNs to be popularized [4], [5]. Usually, batteries and energy collectors provide energy to the devices (e.g., solar panels). Batteries need to be recharged or replaced, and there are several challenges to be overcome regarding energy collectors [6]. Many authors consider energy efficiency one of the most critical design factors for extending network life and reducing maintenance costs [7], [8].

Considering that the energy consumption of WSN nodes is a central issue, this work presents a new model to analyze the energy consumption of end devices from a stochastic point of view. The objective is to offer more subsidies for evaluating networks formed by this type of equipment to make it possible to assess not only the total consumption or the mean consumption but also the probability intervals of drained energy.

The findings should make an essential contribution to the field of WSNs, more specifically, to the engineering and design of this type of network. In this regard, the authors could not find any other studies using the approach proposed in this work.

The remainder of the paper is organized as follows: Section II presents the related work and the main contributions of this research. The theoretical background is presented in Section III. In Section IV, the theoretical model is presented. In Section V, the demonstration and analysis of the model is presented with data collected from simulation. Finally, Section VI gives the conclusions and identifies areas for further research.

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## II. Related Work and Contributions

The models proposed in the literature study different technology, topologies, and device configurations. This section summarizes key related research to this work.

Measurement-based models were discussed: (1) Potsch [9] made a low-cost IoT energy meter and developed two qualitative case studies that revealed system stability and validated the circuit's measuring range; (2) Singh et al. [10] analyzed the current consumption of LoRaWAN, DASH7, Sigfox, and NB-IoT devices. The results showed that LoRaWAN and DASH7 surpassed Sigfox and NB-IoT in the assessed situations. (3) Michelinakis *et al* [11] examined NB-IoT device energy consumption and compared the results to network operator impressions to quantify configuration parameter effects. The authors conclude that modules, operators, signal quality, power saving features (RAI, eDRX), and packet size affect energy use; (4) The model proposed by [12] considers routing, sensing, and transmitting operations to calculate node energy consumption of Texas Instruments CC2530 SoC.

Network behavior and larger amount of network nodes were also modeled: (1) Bouguera [13] presents a LoRaWAN energy model. The models allow hardware and software decisions to be evaluated. Trade-offs have been identified among distance, spreading factor, and transmission power. (2) In [14], two approaches are proposed to calculate and optimize the power consumption of IEEE 802.15.4 IoT devices. (3) Using the Qualnet simulator, Das [15] evaluates the energy usage of AODV, DSR, a general consumption model, MicaMote, and MicaZ models. It has been noticed from simulation results that AODV uses less electricity than DSR.

Advances in modeling and analyzing the random behavior of WSNs were addressed: (1) the research presented by [16] provides a stochastic WSN energy model based on Markovian decision processes that cover energy usage and transition costs. (2) Nguyen [17] created stochastic motes consumption methods based on Random Walk routing. Simulation results demonstrate the model balances energy optimization to improve WSN efficiency. (3) Zhang and Li [18] created a paradigm to improve duty cycle state switching. Performance research using a state transition model (full-active, semi-active, sleep mode) has been accomplished. The results include the analysis of energy usage, average latency as a function of data production rate, and network data throughput. (4) Rahimifar [19] proposed a software-defined IoT network with energy prediction using Markov models. (5) A circuit for measuring energy usage with the INA219 sensor and a probabilistic approach using Markov chains to represent point-to-point and star networks were also constructed by [13]. (7) Lages [20] uses Petri Networks to estimate the energy consumption of LPWAN-based IoT systems. Model validation was done with node experiments.

Although the cited studies provide interesting techniques for modeling and evaluating the power consumption of WSNs, it has been noticed that an analysis that considers the probability distribution function (PDF) and the maximum and minimum expected values of the network's drained energy can complement those results. These studies evaluate energy performance based on average or total energy usage. Thus, the PDF can be utilized to characterize the problem better and demonstrate the statistical significance of estimated parameter values. Besides, the expected maximum and minimum of a random variable permit calculating the chance that an event falls within a specific interval, therefore removing outliers from the analysis if necessary. Consequently, the primary contribution of this study is showing that the energy consumption of a WSN can be modeled as a random variable with its respective PDF and maximum and minimum limits. The importance of doing so is summarized in Figure 1, which presents a sketch showing the energy consumption of all nodes of three networks with the same mean but different probability distribution shapes. Supposing all network nodes consume the same power amount, the probability distribution is best described with the Dirac delta function, $\delta(e-\mu)$. On the other hand, the network nodes can consume different energy values that can be more or less scattered. Another contribution is that the proposed theoretical model is adaptable enough to allow the network designer to investigate networks with arbitrary modes of operation.

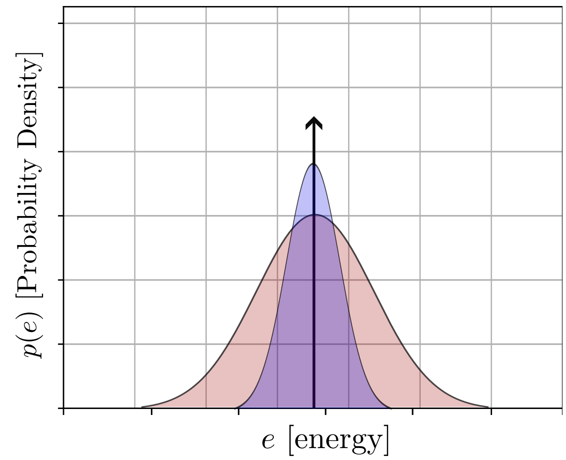


Fig. 1. Three PDFs that have the same mean but different shapes.

## III. Theoretical Background

### A. LoRa and LoRaWAN

LoRa is a technology that creates communication links between EDs and gateways (GWs). LoRaWAN, on the other hand, is the name given to the protocol that specifies the upper layers managing, mainly, access to the medium [21].

In the physical layer, M-ary Chirp Spread Spectrum (MCSS) modulation allows range increase. The choice of the SF (spreading factor) values influences interference, channel effects, and the Doppler effect. High SF values result in low data rates and high ToA (Time on Air), which are the main disadvantages of MCSS [22].

The LoRaWAN standard defines channel access based on the ALOHA protocol and the choice of transmission channels [23]. Additionally, the protocol defines three classes of devices, A, B, and C [24]. Class A devices provide two downlink reception slots, RX1 and RX2. Class B allows several reception slots, which requires synchronization between EDs and GWs. Devices with only one continuously open reception window are Class C. The LoRaWAN specification also determines two types of transmission mode: the confirmed mode, in which the network server (NS) transmits acknowledgment packets (ACK) towards the EDs in response to each transmitted packet; in unconfirmed mode, NS does not send any ACK packets [25]. This work presents results using both Class A devices and confirmed mode.

EDs send packets to GWs, which then send them to the NS. The GWs communicate with the NS via IP connections such as WiFi, Ethernet, or xG (e.g., 3G, 4G, 5G, 6G). The NS is the central point of the network and is responsible for discarding duplicate messages, sending acknowledgment messages, and managing node parameters. The clustering of all devices forms a star of stars topology [26].

### B. Energy Consumption

It has been found in several works that, in general, the energy consumed during an operation cycle can be described according to Equation 1 [10], [13], [27],

$$E_c = \sum_{k=1}^{M} e_k = \sum_{k=1}^{M} t_k \overline{\psi_k}. \tag{1}$$

The total operating cycle energy is given by the sum of the energy consumed in each state wherein $M$ is the number of states, and $e_k$ is the energy consumed during $k$-th state. Considering only the physical aspects, $e_k$ can be obtained by the product of the average power, $\overline{\psi_k}$, and $t_k$ which represents the time interval.

For LoRaWAN EDs working in class A, the energy consumed during an operation cycle is given by the energy spent in uplink transmission, $\mathrm{RX}$ delay 1, downlink reception in

RX1, RX delay 2, downlink reception in RX2 and sleep until the next uplink transmission,

$$E_c = E_{UL} + E_{RX_{Delay1}} + E_{DL_{RX1}} + E_{RX_{\text{Delay2}}} + E_{DL_{RX2}} + E_{\text{Sleep}} = VI_{TX}T_{TX} + VI_{RX_{\text{Delay1}}}T_{RX_{\text{Delay1}}} + VI_{DL_{RX1}}T_{DL_{RX1}} + VI_{RX_{\text{Delay2}}}T_{RX_{\text{Delay2}}} + VI_{DL_{RX2}}T_{DL_{RX2}} + VI_{\text{Sleep}}T_{\text{Sleep}}, \quad (2)$$

in which $V$ represents the voltage, $I$ is the current, and $T$ is the time spent in each state. The time passed in transmission and reception, $T_{TX}$ and $T_{RX}$, are equal to the ToA of the packet.

The NS controls the transmission parameters, the SF, and the transmission power ($P_t$) of the EDs using the Adaptive Data Rate (ADR) algorithm. The ADR optimizes the transmission rate and reduces power consumption. The LoRaWAN standard does not specify it, but most NSs follow the proposal of Semtech [28]. The NS collects the Signal Noise Ratio (SNR) values of the uplink (UL) packets, how many GWs received each *uplink* packet and returns $P_t$ and SF the EDs should use to transmit data.

## IV. Theoretical Model

In this work, an approach to model energy consumption is proposed based on the fact that the energy consumed by any sensor node of the network in a state $k$ is a random variable. In LoRaWAN networks, for example, $P_t$ and SF variation is dominated by ADR algorithm actuation as well as the duration of the reception windows. Therefore, the total energy consumed per cycle, $E_c$, is a random variable.

Real IoT devices have limits on the minimum and the maximum power consumption. A well-designed network must be configured so that the nodes' energy consumption values per cycle must be close to the minimum consumption. Ideally, all consumption values should be equal to the minimum, but, in practice, this is impossible due to random factors that cause variation in power consumption. One possible classification of the network regarding energy consumption is proposed as follows:

1) ideal WSN, if all nodes consume the minimum energy. The most suitable PDF is the impulse function located at the minimum limit, $\delta(e - E_{\min})$;
2) well-designed WSN, if the consumption values are more likely to be close to the minimum. Right-skewed distributions are more suitable in this case;
3) regular WSN, if the mean consumption is around $(E_{\max} + E_{\min})/2$;
4) bad-designed WSN, if the consumption values are more likely to be close to the maximum. Left-skewed distributions are more suitable in this case;
5) worst case, if all nodes consume the maximum energy. The most suitable PDF is the impulse function located at the maximum limit, $\delta(e - E_{\max})$.

A variety of PDFs can be used to model the problem. In this work, one possible formulation is presented, considering that the devices are configured so that the consumption of all states tends to the minimum limit forming a well-designed WSN. Subsection IV-A presents the derivation of the probability distribution of the total energy consumed per cycle, and Subsection IV-B the maximum and minimum energy consumption PDFs.

### A. Probability Distribution of the Energy Consumed per Cycle

In the case of LoRaWAN networks, the time varies mainly due to the transmission and reception window size changes caused by SF choice, which changes the ToA. Also sleep time and idle time also vary due to medium access randomness and reception windows usage. Although both the time and power can be modeled as random variables, this work presents the analysis with the power kept fixed while the time is considered random since it is assumed that the uncertainty is dominated by time variation.

The typical probability distribution for the time interval between events is the exponential distribution given by [29]

$$p_k(t) = \alpha_k \exp[-\alpha_k t]u(t) \quad (3)$$

in which $\alpha_k = 1/\lambda_k$ is the inverse of the time average, $\lambda_k$, and $u(t)$ is the unit step function. One way to obtain the PDF of the energy consumed per cycle is to convolve the PDFs of each state. The result of this operation is given by

$$p_E(e) = \sum_{i=1}^{M} \frac{\beta_1 \cdots \beta_n}{\prod_{\substack{j=1 \\ j \neq i}}^{M} (\beta_j - \beta_i)} \exp[-\beta_i e]u(e), \quad (4)$$

in which $\beta_k = \alpha_k/\overline{\psi_k}$. Demonstration of how to obtain Equation 4 is presented in Appendix A.

### B. Probability Distribution of the Maximum and Minimum Energy Consumption

Maximum and minimum power consumption probability distributions establish theoretical power consumption limits. The min and max operators are helpful for evaluating pessimistic and optimistic scenarios of energy consumption [30]. If $E_1, \ldots, E_N$ are random variables referring to energy consumption during an operating cycle, $W = \min(E_1, \ldots, E_{N_w})$ and $Z = \max(E_1, \ldots, E_{N_z})$ can be used to estimate the highest and lowest consumption of nodes in a WSN.

Assuming the independence of the $N_z$ random variables that represent the consumption per cycle of the sensor nodes, the generic expression for the PDF of the maximum is given by [30]

$$p_Z(z) = \sum_{i=1}^{N_z} p_{X_i}(z) \prod_{\substack{j=1 \\ j \neq i}}^{N_z} P_{X_j}(z). \quad (5)$$

To obtain the PDF of $Z = \max(E_1, ..., E_N)$, the PDF and the CDF (cumulative distribution function) of the consumed energy are substituted in Equation 5.

On the other hand, the generic PDF of $W$ for $N_w$ random variables is given by [30]

$$p_W(w) = \sum_{i=1}^{N_w} p_{X_i}(w) \prod_{\substack{j=1 \\ j \neq i}}^{N_w} [1 - P_{X_j}(w)]. \quad (6)$$

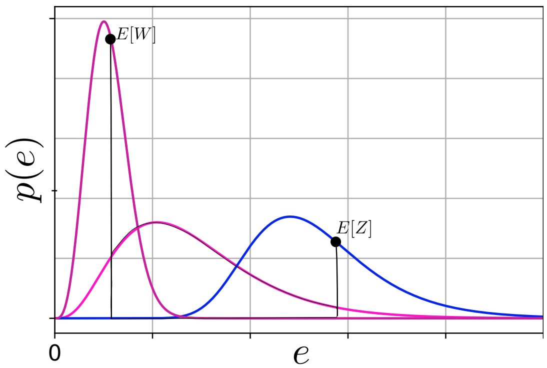


Fig. 2. Energy consumption PDF curve, $p_E(e)$; minimum energy consumption PDF curve, $p_W(w)$; and maximum energy consumption PDF curve, $p_Z(z)$.

The PDF of $W = \min(E_1, ..., E_N)$ is also obtained by substituting the PDF and the CDF in Equation 6. Due to the mathematical complexity, the probability density and the expected values of the maximum and minimum consumption were obtained numerically. Figure 2 presents the PDFs of the maximum, the minimum, and the derived power consumption. Expected values ($E[W]$ and $E[Z]$) represent the theoretical limits.

### C. Technology Minimum and Change of Basis

Naturally, physical variables are bounded by finiteness or causal relationships in the system. Real radios have limits regarding the minimum and maximum energy consumption intrinsic to the construction of the device hardware, the firmware, and configuration parameters used for each application. Therefore, to apply the model, it is necessary to consider the minimum threshold of the device, which is non-zero for most cases. Because the derivation of Equation 4 does not consider this limit, the change of basis can be used, which converts every assertion given in terms of coordinates of one basis into another. The method applied in this work consists of the steps:

1) Calculate the mean energy consumption of each state considering all nodes of the network;
2) Convert every value of $\beta_k$ from the original basis (origin is $(E_{c_{\min}}, 0)$) to the new basis (origin is $(0, 0)$):

$$\beta_k^{'} = 1/(\mu_k - (\mu_k / \sum_{i=1}^{M} \mu_k) E_{c_{\min}}), \tag{7}$$

wherein $\mu_k$ is the mean energy consumption of state $k$;
3) Calculate the PDF of $E_c$ in the new basis, $p_E(e)^{'}$;
4) Convert $p_E(e)^{'}$ to the original basis:

$$p_E(e) = p_E(e - E_{c_{\min}})^{'}. \tag{8}$$

## V. Case Study with LoRaWANSim

LoRaWANSim is a free and open-source simulator developed in MATLAB to evaluate the behavior of the medium access and physical layer of LoRaWAN networks. However, it does not consider the energy consumption in the Processing and I/O modes, although the theoretical model proposed in this work has the flexibility of making it possible for the network designer to study as many states as needed. Tests and comparisons with analytical models and experimental results validated the simulator. The main inputs of the simulator are the arrangement of nodes, the propagation model, and a set of parameters that determine the behavior of the protocol stack layers. As outputs, the simulator returns some metrics: (1) the uplink delivery rate (ULDR) refers to the packets transmitted by the EDs and received by the GW; (2) the performance of the total and mean energy consumption of each ED is estimated, considering the states of transmission, reception, idle, and low consumption modes of operation [27].

### A. Simulation Inputs

In all scenarios, all EDs were placed randomly in a circular area with a 4 km radius. Table I contains the list of the parameters used as input to the simulator. Six scenarios were established to analyze the effects of the transmission period, the environment, and the number of gateways. The number of EDs equals 200 for all cases as the circular area radius is 4 km. Except for the propagation model parameter, the period between transmissions, and the number of gateways and end-devices, all other parameters are the same as in [27].

A comparison of communication in free space and space with obstacles is presented in Subsection V-B2. Log-distance path loss model is appropriate for that being an extension of the free space model. This model equation replaces the original Okumura–Hata model of the published simulator because it is widely utilized with fewer parameters and less complexity. Therefore, it can be used to forecast the propagation loss for various situations [31]. The formulation of the model is given by

$$P_r = P_t + G_t + G_r - 10n \log(d), \tag{9}$$

in which $P_r$ is the received signal power and $P_t$ is the transmission power. The parameters $G_t$ and $G_r$ are the gains of the transmitting and receiving antennas, and $d$ is the distance between the devices.

### B. Simulation Results

This subsection presents the simulation results. The performance outputs are summarized in Table II.

*1) Effect of the Transmission Period:* Energy consumption histograms and associated theoretical PDF curves are shown in Figure 3 for two different scenarios. In both cases, the EDs exchange data with a single GW in a free space environment ($n = 2.0$) for $T = 30$ s and $T = 300$ s. As was to be predicted, transmission interval lengthening results in reduced power consumption. In the case of $T = 300$ s, simulation results more closely match the theoretical probability density function (PDF). Energy consumption observations for the $T = 30$ s scenario are slightly above the convolution of exponentials probability distribution. Greater concurrence for the medium explains this by high interference and collisions, pushing energy consumption numbers further from the minimum. According to Table II, the ULDR decreases as the

TABLE I
INPUT PARAMETERS FOR THE CHOSEN SCENARIOS.

| Parameter | Description | Value |
|---|---|---|
| $N$ | Number of nodes | {200} |
| $G$ | Number of gateways (GWs) | {1, 15} |
| $R$ **[km]** | Area radius | 4 |
| $P_T^{ED}$ **[dBm]** | ED transmission power | {14, 12, 10, 8, 6, 4, 2} |
| $G_A^{ED}$ **[dBi]** | ED antenna gain | 5 |
| $P_T^{GW}$ **[dBm]** | GW transmission power | 16 |
| $G_A^{GW}$ **[dBi]** | ED antenna gain | 5 |
| $\sigma$ **[dB]** | Shadowing parameter | 3 |
| $n$ | Propagation model parameter | {2.0, 4.0} |
| $CR$ **[bits/s]** | Coding rate | 1 |
| $BW$ **[kHz]** | Bandwidth | 125 |
| $DC$ [%] | Duty cycle restriction | 100 (no DC restriction) |
| $T$ **[s]** | Period between transmissions | {30, 300} |
| $H$ **[Boolean]** | Header Presence? | 1 (true) |
| $L_p$ **[bytes]** | Message preamble length | 8 |
| $B_{UL}$ **[bytes]** | Uplink packet Size | 20 |
| $B_{DL}$ **[bytes]** | Downlink packet Size | 20 |
| RX1DROffset | Offset between SF of transmission and RX1 reception window | 0 (no offset) |

TABLE II
SUMMARY OF THE SCENARIOS AND THEIR RESPECTIVE PERFORMANCE OUTPUTS ANALYZED IN THIS WORK.

| Scenarios | | | | ULDR [%] | $\mu$ [mJ] | $E[E\_c]$ [mJ] | $E_{c_{\min}}$ [mJ] | $E_{c_{\max}}$ [mJ] | $E[W]$ [mJ] | $E[Z]$ [mJ] |
|---|---|---|---|---|---|---|---|---|---|---|
| ID | $T$ | $n$ | GW | | | | | | | |
| 1 | 30 | 2 | 1 | 63.50 | 140.75 | 140.61 | 97.39 | 194.22 | 100.44 | 298.72 |
| 2 | 30 | 4 | 1 | 68.34 | 152.12 | 151.58 | 97.41 | 224.59 | 101.71 | 277.89 |
| 3 | 30 | 4 | 15 | 92.27 | 113.83 | 113.83 | 97.39 | 161.85 | 98.32 | 184.90 |
| 4 | 300 | 2 | 1 | 93.58 | 105.67 | 105.67 | 97.94 | 125.42 | 98.36 | 139.48 |
| 5 | 300 | 4 | 1 | 94.38 | 112.20 | 112.20 | 98.00 | 225.22 | 98.99 | 171.41 |

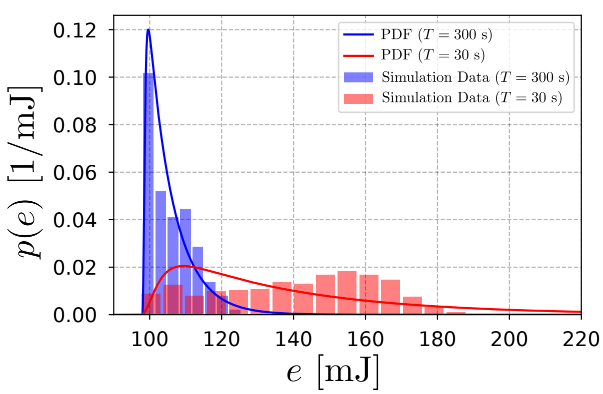


Fig. 3. Energy consumption histograms for $T = 30$ s and $T = 300$ s, and their theoretical PDF curves.

power consumption increases when $T = 30$ s. It is possible to notice that the verified minimum and maximum collected from the simulation are different from the expected minimum and maximum. However, this is to be expected since the ADR algorithm can optimize the network in such a way that the energy consumption of most nodes is closer to the minimum.

*2) Effect of the Environment:* Figure 4 displays the theoretical PDF curves related to the histograms of energy consumption for two separate cases regarding the log-normal propagation model exponent. The EDs communicate with a single GW in a free space environment ($n = 2.0$) and inside a space with obstacles ($n = 4.0$). The period between transmissions is $T = 300$ s in both cases. Deploying the nodes in a hasher region has the expected effect of increasing energy use. The theoretical probability density function is more closely matched by the simulation findings when $n = 2.0$. The observed energy consumption for the $n = 4.0$ scenario is slightly higher due to smaller SNR, so the ADR algorithm increases the transmission power of the nodes. The ULDR values for both cases are very close, indicating that the environment has a greater effect on the energy than the delivery rate. Some outliers were detected, as can be verified by comparing $E[Z]$ to $E_{c_{\max}}$. It is the responsibility of the network designer to select the value to truncate the PDF from $E[Z]$, $E[W]$, $E_{c_{\max}}$ and $E_{c_{\max}}$ that best suit the application even though it is highly recommended to remove the rare values out of the analysis.

*3) Effect of the Number of Gateways:* In these two scenarios, the nodes are distributed for communication with both 1 and 15 gateways placed randomly. Theoretical PDF curves associated with energy consumption histograms are shown in Figure 5. Both examples use $T = 30$ s and $T = 4.0$ to evaluate intense medium concurrence and lower SNR. Energy consumption and ULDR improve as a result of more GWs. The simulation results align with the theoretical probability density function when $GW = 15$. More gateways cause the WSN performance to be improved than expected, although the deployment cost increases.

## VI. FINAL REMARKS

This work presents a theoretical stochastic model to calculate the probability ranges of WSN energy consumption.

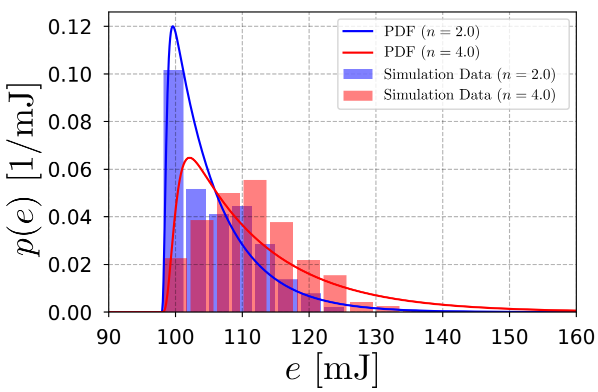


Fig. 4. Energy consumption histograms for $n = 2.0$ and $n = 4.0$, and their theoretical PDF curves.

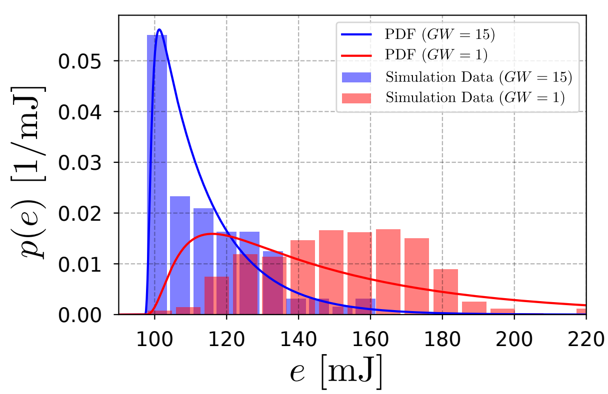


Fig. 5. Energy consumption histograms for $GW = 1$ and $GW = 15$, and their theoretical PDF curves.

The theoretical PDF curves were produced, and the distributions related to the maximum and minimum consumption were derived, allowing the network designer to evaluate the power consumption distribution of the entire network. The PDF gives an entire picture of WSN power consumption as the maximum and minimum expected and observed values allow the energy dispersion analysis and outliers removal. The model was applied to evaluate six different scenarios in a more detailed way. It has been shown that when the period between transmissions is reduced, obstacles are present and few gateways are used in a harsh environment, the mean energy increases as well as the dispersion of the energy values. Also, for the analyzed scenarios, the energy consumption of all network nodes are between expected maximum and minimum theoretical prediction, except for Scenario 5, in which outliers were detected. In addition, it is possible to conclude that Scenarios 3 and 4 are better designed than Scenarios 1, 2 and 5. The main contribution of the paper is the developed approach which can be used to evaluate WSN with more detail. Based on the results, the authors expect WSN designers to have more subsidies to examine the performance of different applications in this area of knowledge using the proposed methodology. Other probability distributions can be used to model the power consumption depending on the random dominant mechanism and will be investigated in a future work.

## Appendix

### Appendix A – Demonstration of Equation 4

The energy consumed in each state is given by

$$e_k = t_k \overline{\psi_k}. \tag{10}$$

Applying a PDF transform [32],

$$p_k(e) = \frac{f_k(t)}{|\frac{de}{dt}|} = \frac{\alpha_k \exp[-\alpha_k e/\overline{\psi_k}] u(e/\overline{\psi_k})}{\overline{\psi_k}},$$

making $\beta_k = \alpha_k/\overline{\psi_k}$ and using the property of the unit step function, the PDF related to the consumption in a state $k$ becomes

$$p_k(e) = \beta_k \exp[-\beta_k e] u(e). \tag{11}$$

Let $e_1, e_2, ..., e_M$ be independent random variables with exponential distribution and let $E_c$ be the sum of these variables denoted by

$$E_c = \sum_{k=1}^{M} e_k. \tag{12}$$

It is possible to obtain the distribution of $E_c$ with the convolution of the PDFs of $e_k$ [33],

$$f_E(e) = f_1(e) * f_2(e) * \cdots * f_M(e). \tag{13}$$

By induction, it can be demonstrated that Equation 4 is the probability density function for $M$ states.

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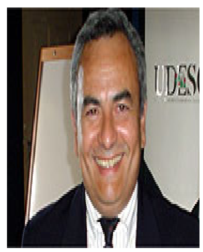


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