# Best Response Dynamics for Collective Route Optimization

M. de L. Angulo-Dominguez, P. Mejia-Alvarez, R. Menchaca-Mendez, and A. Yee-Rendon

***Abstract*—Urban traffic congestion remains a critical challenge for modern cities, impacting travel efficiency, environmental sustainability, and quality of life. This paper introduces the Collective Optimization Scheme (COS), a collaborative routing framework that integrates Best Response Dynamics with Dijkstra's algorithm to promote cooperative decision-making among drivers. Unlike traditional navigation systems that optimize routes individually, COS computes routes that account for prevailing congestion conditions and aim to minimize the total travel time across all trips. The proposed approach is evaluated through extensive simulations on real-world urban maps, demonstrating substantial reductions in travel time particularly under low to moderate congestion levels. These results highlight COS as a scalable and effective strategy for sustainable congestion management and improved urban mobility.**

**Link to graphical and video abstracts, and to code:**
**https://latamt.ieeer9.org/index.php/transactions/article/view/10203**

***Index Terms*—Best response dynamics, collaborative routes, Nash equilibrium, shortest path problem, traffic behavior modeling , traffic vehicular congestion.**

## I. Introduction

URBAN traffic congestion is not merely a transportation issue but an urgent urban crisis with far-reaching consequences. It impacts economic productivity, environmental sustainability, and the quality of life of urban residents [2], [22]. Rapid urban growth and the rising demand for mobility continue to place significant strain on road infrastructures [25], resulting in increased congestion, prolonged travel times, and higher emissions. These challenges underscore the urgent need for effective traffic management strategies and sustainable mobility solutions [4].

Navigation applications such as Google Maps, Waze, and TomTom employ algorithms that model road networks by assigning costs to each connection based on factors such as travel time, distance, tolls, and energy consumption. While these systems effectively assist users in navigating cities, they face limitations in handling dynamic conditions such as traffic priorities, bottlenecks, accidents, and road closures [21]. Despite significant technological advances, further improvements are needed to enhance their adaptability to constantly evolving urban environments.

Completely eliminating vehicular traffic from cities is a complex and long-term goal; however, targeted strategies can substantially reduce its negative effects. Serdar [25] demonstrates that generating routes aimed at achieving a social optimum while still allowing for a degree of individual choice can outperform purely selfish routing strategies in terms of travel time. Similarly, Lanning and Large [10], [13] emphasize that individualistic route generation often leads to many drivers selecting the same path, causing localized congestion. Based on the conclusions of Cabannes [5], Serdar [25], and Lei [15], the use of cooperative routing strategies can effectively reduce travel times for multiple trips and mitigate urban congestion.

Building on this premise, we propose encouraging collaborative route selection among drivers. Our approach applies game theory [1], specifically the Best Response Dynamics (BRD) strategy, to balance individual self-interest with collective efficiency, leading to a Nash equilibrium in which no driver has an incentive to unilaterally change routes. This equilibrium promotes a more balanced use of road network capacity and contributes to congestion reduction. The proposed *Collective Optimization Scheme* (COS) minimizes total travel time for a set of trips by representing the city as a graph, assigning edge weights via a congestion function, and computing routes collaboratively.

The COS algorithm employs BRD to iteratively adjust drivers' route choices in response to others' decisions, aiming for traffic equilibrium. Each driver adapts their path based on current traffic conditions shaped by collective behavior. Dijkstra's shortest path algorithm [6] is incorporated to efficiently compute optimal routes while accounting for congestion dynamics. Experimental results demonstrate that collaborative routing significantly reduces travel times, particularly under low to moderate congestion, providing strong evidence of COS's potential to enhance traffic efficiency in urban settings.

The main contributions of this work are as follows:

- Introduction of a collective optimization scheme for generating collaborative routes across a set of trips in a road network.
- Development of an algorithm that integrates Best Response Dynamics with Dijkstra's algorithm to produce cooperative routes.

The algorithm generates both collaborative and individual routes, which are compared in terms of travel time using the proposed congestion function. This allows for an evaluation

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of efficiency and behavioral patterns under varying traffic conditions. Additionally, the SUMO (Simulation of Urban Mobility) platform is employed to visualize route performance, enabling detailed analysis of traffic dynamics and the impact of collaborative routing on congestion reduction.

The remainder of this paper is organized as follows: Section I examines the theoretical background and related work. Section II formalizes the problem. Section II-C describes the proposed solution. Section III details the methodology. Section IV reports experimental results. Section V discusses related work, and Section VI concludes the paper.

## II. Problem Formulation

This study addresses the problem of collaboratively optimizing vehicle routing in an urban road network modeled as a weighted, directed graph. The objective is to minimize the total travel time for all vehicles by considering the aggregate effect of individual routing decisions on the network.

The process begins with the construction of the road network graph Fig. 1, where intersections are represented as vertices and streets as directed edges. Each edge is assigned a weight that reflects both the physical characteristics of the street segment and prevailing traffic conditions.

A trip is defined as an origin-destination(OD) pair. The set of trips collectively represents the mobility demand of all agents in the system. Each trip is assigned a path through the network.

*Definition 2.1 (Road Network):* A road network is a directed, weighted graph $G = (V, E, w)$, where $V$ is the set of vertices representing intersections or endpoints of road segments, $E \subseteq V \times V$ is the set of directed edges representing road segments, and $w : E \to \mathbb{R}^+$ is a weight function assigning a positive cost to each edge.

To construct the graph, a specific urban area was selected and converted into a directed network using OpenStreetMap [9] data , as shown in Fig. 1. In this representation, each edge corresponds to a street segment, and each vertex represents a street intersection. Edge weights are determined by the segment's physical and operational attributes, such as length, number of lanes, and estimated travel time.

The resulting graph structure, highlighted in the pink section of Fig. 1, provides a concrete example of the network representation adopted in this study.

### A. Graph Description and Example

The road network is modeled as a directed, weighted graph, with vertices, edges, and their associated weights defined as follows.

*Definition 2.2 (Graph):* A directed, weighted graph $\boldsymbol{G}$ is defined as follows:

$$\boldsymbol{G = (V, E, w)} \tag{1}$$

where $\boldsymbol{V}$ is a set of vertices, $\boldsymbol{E}$ is a set of edges and $\boldsymbol{w}$ is their associated weights for each edge.

Fig. 1 illustrates an example of the corresponding graph representation, while the subsequent equations detail each component of the graph:

$$\boldsymbol{V = \{v_1, v_2, \ldots, v_n\} = \{I, II, III, IV\}} \tag{2a}$$

$$\boldsymbol{E = \{e_1, e_2, \ldots, e_m\} = \{a, b, c, d, e, f, g, h\}} \tag{2b}$$

$$\boldsymbol{w : E \to \mathbb{R}_0^+, \quad w = \{6, 6, 8, 1, 4, 6, 4, 5\}} \tag{2c}$$

Each edge $\boldsymbol{e_i} \in \boldsymbol{E}$ is assigned a non-negative weight $\boldsymbol{w(e_i)}$, representing the travel time (in seconds), required to traverse the corresponding segment under the congestion model.

*Definition 2.3 (Trip):*
A trip $\boldsymbol{t_i}$ is defined as an origin-destination pair as follows:

$$\boldsymbol{t_i = \{o_i, d_i\}, \quad o_i, d_i \in V} \tag{3}$$

where $\boldsymbol{o_i}$ is the origin vertex and $\boldsymbol{d_i}$ is the destination vertex.

*Definition 2.4 (Set of Trips):* The set of trips is denoted by:

$$\boldsymbol{T = \{t_1, t_2, \ldots, t_n\}} \tag{4}$$

containing all trips in the system. where $\boldsymbol{n}$ is the total number of agents, and each trip $\boldsymbol{t_i} \in \boldsymbol{T}$ is associated with a unique path through the network.

*Definition 2.5 (Path):* A path $\boldsymbol{r}$ for a given trip is defined as an ordered sequence of vertices connecting its origin and destination, expressed as

$$\boldsymbol{r = [v_1, v_2, \ldots, v_k], \quad v_i \in V,} \tag{5}$$

which represents the route followed by a vehicle through the network.

The edge weights $\boldsymbol{w(e)}$ are determined by a congestion function that models traffic behavior and reflects the saturation level of each segment. These weights are essential for route computation in the proposed collaborative routing framework.

*Definition 2.6 (Congestion Function):* The congestion function models the nonlinear relationship between traffic load and travel time. Following [27], congestion is represented through a sigmoid function that captures the rapid increase in travel time as the load approaches the road segment capacity. Accordingly, the congestion function is defined as

$$\boldsymbol{f(x) = \frac{k_1}{1 + e^{-k_2(x - k_3)}} + k_4,} \tag{6}$$

where the parameters incorporate both physical and traffic-related characteristics of each road segment.

The congestion level in (6) is expressed in terms of saturation (see Fig. 2), defined as a dimensionless ratio in the range **0–1**, representing the proportion of vehicles (congestion percentage) relative to the maximum load capacity of a road segment.

Parameter definitions:

- $\boldsymbol{x}$: current number of vehicles on the edge.
- $\boldsymbol{k_1}$: range of additional delay caused by congestion, defined as:

$$\boldsymbol{k_1} = \frac{\text{length}_{\text{street}}}{\text{vel}_{\text{min}}} - \frac{\text{length}_{\text{street}}}{\text{vel}_{\text{max}}}, \tag{7}$$

  which represents the difference between travel times at minimum and maximum speeds.

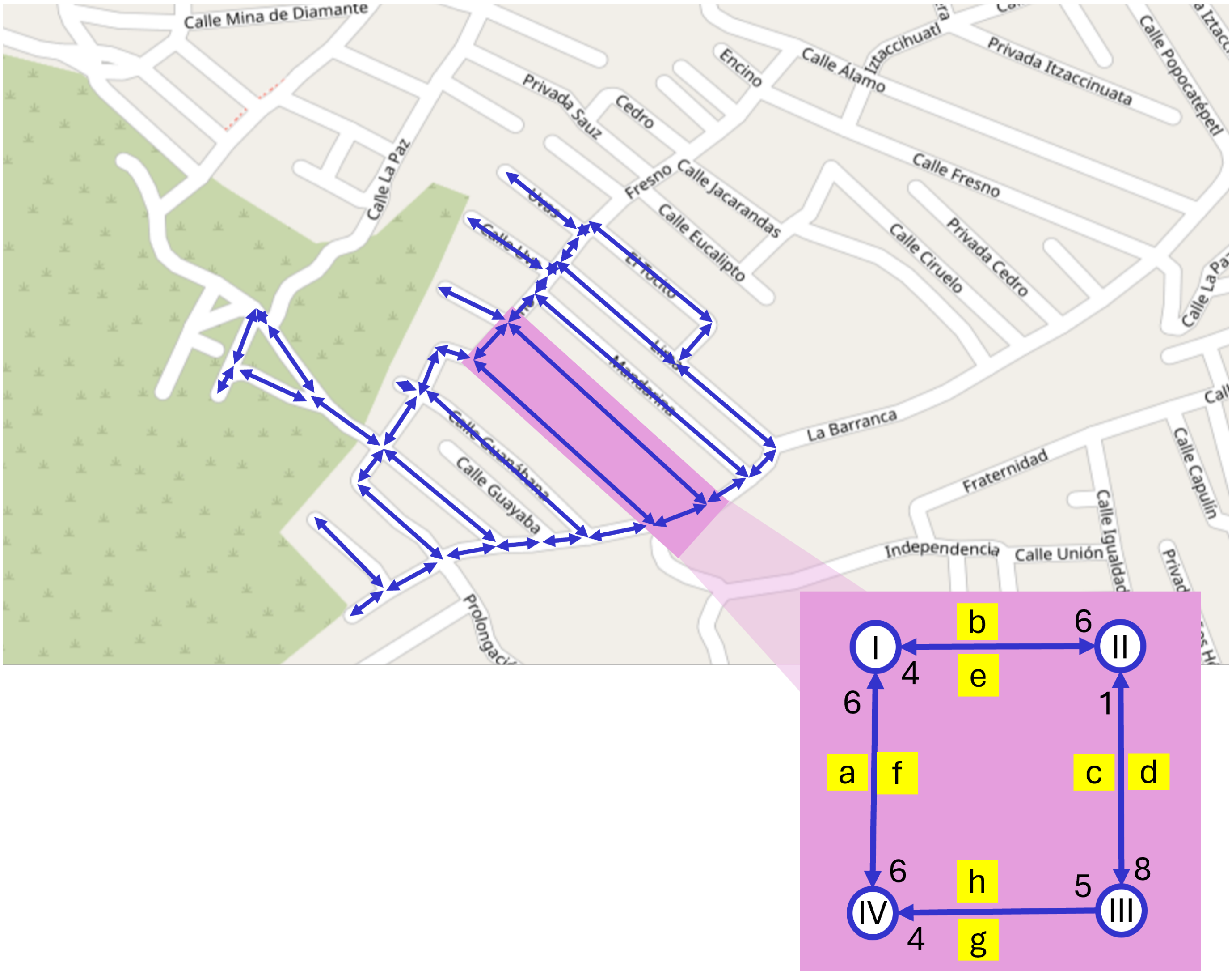


Fig. 1. Representation of the road network as a directed graph. The main map shows the selected urban area with its corresponding graph structure. The inset presents a simplified example with labeled edges and their associated weights.

- $k_2$: steepness of the sigmoid curve, defined as:

$$k_2 = \frac{M + k_3}{\text{load}_{\text{max}}}, \tag{8}$$

where $M$ is a dimensionless smoothing factor (typically set to $10$) and $\text{load}_{\text{max}}$ denotes the maximum vehicular load of the segment. The parameter $M = 10$, adopted from [27], controls how sharply the sigmoid congestion curve transitions around the maximum load, ensuring a smooth yet sufficiently steep transition and numerically stable behavior.
- $k_3$: midpoint of the congestion curve, corresponding to the inflection point, given by:

$$k_3 = \frac{\text{load}_{\text{max}}}{2}. \tag{9}$$

- $k_4$: baseline free-flow travel time, defined as:

$$k_4 = \frac{\text{length}_{\text{street}}}{\text{vel}_{\text{max}}}, \tag{10}$$

which corresponds to the travel time under free-flow conditions.

*Definition 2.7 (Maximum Vehicular Load of an Edge):* The maximum vehicular load $\text{load}_{\text{max}}$, used in (8) and (9), is defined as the maximum number of vehicles that can occupy a road segment under ideal conditions, expressed as:

$$\text{load}_{\text{max}} = \frac{\text{length}_{\text{street}}}{\text{Size}_{\text{vhc}}} \times \text{num}_{\text{lanes}}, \tag{11}$$

where $\text{length}_{\text{street}}$ is the segment length (m), $\text{Size}_{\text{vhc}}$ is the average space occupied per vehicle (m), and $\text{num}_{\text{lanes}}$ is the number of lanes.

*Definition 2.8 (Minimum Edge Time):* The minimum traversal time of an edge $x$ at maximum speed $v_{\text{max}}$ is defined as:

$$t_{\text{min}}[x] = \frac{\text{length}[x]}{\text{vel}_{\text{max}}[x]}, \tag{12}$$

which corresponds to free-flow travel conditions.

*Definition 2.9 (Maximum Edge Time):* The maximum traversal time of an edge $\boldsymbol{x}$ at minimum speed $\boldsymbol{v}_{\text{min}}$ is defined as

$$\boldsymbol{t}_{\text{max}}[\boldsymbol{x}] = \frac{\text{length}[\boldsymbol{x}]}{\text{vel}_{\text{min}}[\boldsymbol{x}]}, \tag{13}$$

representing highly congested conditions.

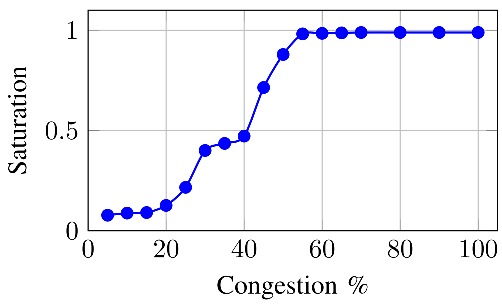


Fig. 2. Behavior of the congestion function (Please see (6)) for a single edge.

Fig. 2 shows the congestion function (6) behavior for a single edge as the number of vehicles increases from 1 to its maximum capacity, corresponding to congestion levels from 5% to 100% in 5% increments.

### *B. Optimization Problem*

The task of determining the shortest path between two points in a graph is a well-known combinatorial optimization problem with applications in transportation, network routing, and logistics. The objective is to identify the most efficient route in terms of distance, travel time, or cost.

In this work, Dijkstra's algorithm is used to determine the shortest path between two vertices, providing an efficient method for computing the optimal route in the absence of congestion effects. To extend this to a collaborative context, we integrate the Best Response Dynamics (BRD) strategy, enabling optimization of the total travel time for a set of trips. This approach accounts for the collective impact of all routing decisions, redistributing traffic to reduce congestion and achieve more balanced network utilization. By considering system-wide effects rather than isolated route choices, this collaborative optimization framework promotes smoother traffic flow and shorter travel times for individual trips.

The optimization objective defined in (14) aims to minimize the total travel time across all trips while accounting for congestion effects on each edge of the network. Specifically, the objective determines the set of routes assigned to all trips such that the aggregated travel cost, computed as the sum of congestion-dependent edge costs, is minimized:

$$\min_{\{r_i\}_{i\in T}} \sum_{i\in T}\sum_{e\in r_i} c_e(x_e) \tag{14}$$

where:

- $T$ is the set of trips,
- $r_i$ is the path assigned to trip $i$,
- $e$ denotes an edge in the road network,
- $x_e$ is the number of vehicles using edge $e$,
- $c_e(x_e)$ is the congestion-dependent travel time of edge $e$.

### *C. Proposed Approach*

Best Response Dynamics (BRD) is a game-theoretic concept describing how individuals sequentially adjust their strategies to maximize their utility based on the actions of others [19]. In a strategic setting, each player selects the strategy that yields the highest payoff given the strategies chosen by the other participants. This iterative process continues until a stable state known as a Nash equilibrium is reached, at which point no player has an incentive to unilaterally deviate from their current strategy [8].

*Definition 2.10 (Game Description and Best Response Dynamics):* Let $s$ be a strategy vector and $i$ a player.

When player $i$ adopts strategy $s$, she/he achieve a utility $u_i(s)$. If she/he switch from her/his current strategy $s_i$ to an alternative $s'_i \in S_i$, her/his utility becomes $u_i(s'_i, s_{-i})$, assuming all other players maintain strategies $s_{-i}$.

A transition from $s_i$ to $s'_i$ is an *improving response* if the following holds:

$$u_i(s'_i, s_{-i}) > u_i(s) \tag{15}$$

where $s_{-i}$ denotes the strategy profile of all players except player $i$.

The *best response* is achieved if the following holds:

$$s^*_i = \arg\max_{s'_i\in S_i} u_i(s'_i, s_{-i}) \tag{16}$$

In distributed systems such as routing networks, there is a strong link between Nash equilibrium and BRD. Once a Nash equilibrium is attained, no single node can improve its outcome through a unilateral change. Stability is thus achieved when every participant follows their best response given the strategies of others [18].

In atomic selfish routing, each participant selects exactly one path from a finite set of options. Each edge $e$ has an associated non-decreasing cost function $c_e : \{1, 2, \ldots, k\} \rightarrow \mathbb{R}^+$, representing the per-player cost as a function of the number of users on that edge. A pure-strategy Nash equilibrium (PNE) occurs when each participant simultaneously selects the path that minimizes their individual cost given the choices of the others [28] [7].

*Definition 2.11 (Pure Nash Equilibrium in Route Decisions):*

BRD models how players iteratively adjust their choices over time. If the current outcome is not a PNE, a player who is not following a best response is selected, and their strategy is updated accordingly. While this process may fail to converge in certain games (e.g., "Rock–Paper–Scissors"), in atomic selfish routing networks each step strictly decreases the value of a potential function as follows:

$$\phi(P_1, \ldots, P_k) = \sum_{e\in E}\left[c_e(1) + c_e(2) + \cdots + c_e(x_e)\right], \tag{17}$$

where $x_e$ is the number of paths $P_i$ containing edge $e$. Since this potential function is bounded below, convergence to a PNE is guaranteed [23].

In summary, BRD offers an effective iterative framework for reaching a PNE [28] in environments where agents act selfishly and discretely. While convergence is not assured in all game types, its guaranteed convergence in atomic selfish routing networks makes it particularly suitable for computing stable equilibria in traffic routing scenarios.

*Definition 2.12 (Best Response Dynamics and Nash Equilibrium):*

In game theory, a best response [20] is the strategy that maximizes a player's utility given the strategies adopted by all other players [30]. Formally, player $i$'s optimal strategy $s_i^*$ belongs to the best-response correspondence $BR(s_{-i})$ if and only if

$$s_i^* \in BR(s_{-i}) \quad \text{iff} \quad \forall s_i \in S_i,\ u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}), \tag{18}$$

where $s_{-i}$ denotes the strategy profile of all players except player $i$.

A *Nash equilibrium* is a strategy profile $s = \langle s_1, \ldots, s_n \rangle$ such that every player's strategy is a best response to the strategies of the others, i.e.,

$$\forall i, \quad s_i \in BR(s_{-i}). \tag{19}$$

where:

- $n$ is the total number of players.
- $s_i$ is the strategy chosen by player $i$.
- $s_{-i}$ denotes the strategies of all players except player $i$.
- $S_i$ is the set of all possible strategies available to player $i$.
- $u_i(s_i, s_{-i})$ is the utility (payoff) obtained by player $i$.
- $BR(s_{-i})$ represents the best response of player $i$.
- $s_i^*$ denotes the optimal strategy for player $i$.

A Nash equilibrium is reached when every player is executing a best response to the others' strategies, meaning no participant can improve their outcome unilaterally. In routing networks, this concept ensures stable traffic patterns where no driver can reduce their travel time without worsening conditions for others.

## III. Methodology

### A. Schematic Representation of the Methodological Process

The Collective Optimization Scheme (COS) operates in two main phases. The *initialization phase*, executed once per map, involves gathering geographic data and constructing the necessary data structures. The *decision-making phase*, repeated for each new trip set, adapts to the specific scenario and generates both individual and collaborative routes.

The process begins by downloading geographic and attribute data from OpenStreetMap, producing a *.osm* file. This file is converted to *.net.xml* format for use in the SUMO simulator, and additional information is extracted to build the road network graph, stored in a *.net* file.

Once the graph is constructed, the initialization phase concludes. In the second phase, the system uses the trip set and the constructed graph to compute individual and collaborative routes, applying traffic-related computations such as congestion functions and travel time evaluation.

COS workflow can be summarized as follows:

1) Construct the road graph and apply initial configurations.
2) Assign edge weights using the proposed congestion function, based on street attributes and an optional static vehicular load.
3) Define the trip set with origin-destination pairs.
4) Compute individual routes and their corresponding travel times.
5) Update edge weights according to the assigned routes.
6) Generate collaborative routes and update the affected edges.
7) Compare travel times between individual and collaborative strategies.

### B. Collective Optimization Algorithm

The Collective Optimization Algorithm (COA) (Please see 1) optimizes multiple routes simultaneously in a directed, weighted graph with dynamic congestion. It integrates a congestion-aware version of Dijkstra's algorithm with Best Response Dynamics (BRD) to achieve convergence toward a collaborative traffic equilibrium.

The algorithm proceeds in three stages:

- Initial Phase: Each trip is assigned an individually optimal route using the congestion-aware Dijkstra algorithm.
- Iterative Refinement: Routes are adjusted iteratively through BRD, where users adapt their paths to updated congestion levels.
- Termination: The process stops when no trip can unilaterally reduce its travel time, reaching a Nash equilibrium.

This integration balances traffic loads across the network, reducing overall travel times and improving system-wide efficiency.

*Step-by-Step Execution of the COA*

The COA can be described in the following steps:

1) Initialization
   - Define $G = (V, E, w)$: directed and weighted graph.
   - Define $T = \{t_1, t_2, \ldots, t_n\}$: set of trips as OD pairs.
   - $c_e$ is the travel time of edge $e$.
2) Initial Route Assignment
   - Compute the shortest path for each trip with Dijkstra's algorithm.
   - Store the resulting routes and travel times in $R$.
3) Congestion Update
   - Update the edge weights of the graph based on the congestion function.
4) Best Response Dynamics
   - Select a user $a \in T$ in order to first al last travel.
   - Recompute route $r'_a$ with updated weights.
   - If $r'_a$ yields a lower travel time, update $R$.
   - Repeat until convergence with the Nash equilibrium (when there are no changes in routes).

**Algorithm 1** Collective Optimization Algorithm (COA)

**Require:** Directed weighted graph $G = (V, E)$, trip set $T = \{t_1, \ldots, t_n\}$ where $t_a = (o_a, d_a)$, congestion functions $\{c_e(\cdot)\}_{e \in E}$

**Ensure:** $R$: set of optimized routes and travel times (Nash equilibrium)

$n \leftarrow |T|$
$R \leftarrow \emptyset$
Initialization and Initial Route Assignment
**for** $i \leftarrow 1$ **to** $n$ **do**
  $r_i \leftarrow Dijkstra(G, o_i, d_i)$
  $\tau_i \leftarrow \sum_{e \in r_i} w_e$ {initial weights $w_e$}
  $R \leftarrow R \cup \{(t_i, r_i, \tau_i)\}$
**end for**
Congestion Update (edge loads and weights)
$L \leftarrow EdgeLoads(R)$ {$L_e$: number of trips using edge $e$}
$w \leftarrow UpdateCongestion(L)$ {$w_e \leftarrow c_e(L_e)$ for all $e \in E$}
**for** $i \leftarrow 1$ **to** $n$ **do**
  Update $\tau_i \leftarrow \sum_{e \in r_i} w_e$ in $R$
**end for**
Best Response Dynamics (first-to-last trip order)
**while** true **do**
  $Nash_{counter} \leftarrow 0$
  **for** $a \leftarrow 1$ **to** $n$ **do**
    {select user in order: first to last trip}
    Retrieve current $(r_a, \tau_a)$ for $t_a$ from $R$
    $r'_a \leftarrow Dijkstra(G, o_a, d_a)$ {computed under current weights $w$}
    {Hypothetical edge-load update for evaluation}
    $L' \leftarrow L$
    $L'_e \leftarrow L'_e - 1$ for all edges $e \in r_a$ {if user $a$ is counted there}
    $L'_e \leftarrow L'_e + 1$ for all edges $e \in r'_a$
    $w' \leftarrow UpdateCongestion(L')$ {$w'_e \leftarrow c_e(L'_e)$}
    $\tau'_a \leftarrow \sum_{e \in r'_a} w'_e$
    **if** $\tau'_a < \tau_a$ **then**
      $R \leftarrow R \setminus \{(t_a, r_a, \tau_a)\} \cup \{(t_a, r'_a, \tau'_a)\}$
      $L \leftarrow L'$ {commit loads}
      $w \leftarrow w'$ {commit weights}
    **else**
      $Nash_{counter} \leftarrow Nash_{counter} + 1$
    **end if**
  **end for**
  **if** $Nash_{counter} = n$ **then**
    break {Nash equilibrium reached: no route changes in a full pass}
  **end if**
**end while**
return $R$

### *C. Convergence of Algorithm*

The proposed COA follows a best-response dynamics procedure in a routing game where each trip selects a path and the cost of a path is the sum of edge costs. Edge weights are updated as a function of the current edge loads, i.e., $w_e = c_e(L_e)$, where $L_e$ denotes the number of trips using edge $e$. Under the standard assumption that each congestion function $c_e(\cdot)$ is non-decreasing with respect to $L_e$, the induced routing game is a congestion game and admits an exact potential function. Consequently, each successful best-response update strictly decreases the potential, preventing improvement cycles. Therefore, the iterative process terminates after a finite number of route changes at a pure Nash equilibrium, which is detected in Algorithm 1 when a full pass over all trips produces no route updates.

### *Algorithm Complexity*

The Collective Optimization Algorithm (COA), presented in Algorithm 1, generates collaborative routes for a set of trips by iteratively applying Best Response Dynamics. Its computational complexity can be analyzed as follows.

*Assumptions and Notation:*

- $G = (V, E, w)$: directed, weighted graph.
- $n = |T|$: number of trips (origin-destination pairs).
- $|V|$ and $|E|$: number of vertices and edges, respectively.

*1. Dijkstra Algorithm Complexity:* Using Python's heapq-based implementation of Dijkstra's algorithm, the time complexity is given by:

$$O\big((|V| + |E|)\log|V|\big), \tag{20}$$

since each vertex is extracted from the priority queue at most once, each edge is processed once, and heap operations incur logarithmic cost.

*2. Initial Route Assignment:* For $n$ trips, where each trip invokes Dijkstra's algorithm once followed by travel-time estimation, the overall computational cost becomes:

$$O\big(n(|V| + |E|)\log|V|\big). \tag{21}$$

*3. Iterative Refinement Phase:* During the Best Response Dynamics (BRD) process, each of the $n$ users may update their route multiple times. In the worst case, each user can perform up to $n$ updates, and every update requires a full path recomputation and congestion recalculation. This leads to a worst-case complexity of

$$O\big(n^2(|V| + |E|)\log|V|\big). \tag{22}$$

*4. Overall Complexity:* By combining the initial route assignment and the iterative refinement phases, the total worst-case computational complexity is therefore

$$\boxed{O\big(n^2(|V| + |E|)\log|V|\big)}. \tag{23}$$

This expression represents an upper bound on the computational cost; in practice, convergence is typically achieved in significantly fewer iterations.

*Algorithmic Considerations:* From the previous analysis, several observations can be derived:

- The overall complexity is quadratic regarding the number of trips and nearly linear with respect to the size of the road network.
- Congestion updates scale approximately linearly with the number of affected edges, assuming local recomputation after each route change.
- The initial route assignment stage can be parallelized across trips, which can significantly reduce execution time in practical implementations.

### D. Application Scenario and System Integration

The methodology is designed primarily for semi-online or offline optimization within municipal traffic control systems or fleet management platforms. COS can be executed periodically to redistribute vehicular load before rush hours. The optimized routes can then be shared with navigation services or autonomous fleet controllers. The system assumes partial compliance from users-simulation analysis shows that COS performance remains stable as long as at least 60% of drivers follow cooperative routes. Non-compliant users are modeled as stochastic perturbations, and their effect on convergence and total travel time is quantified in Section IV.

## IV. Experimental Results

The Collective Optimization Scheme (COS) is designed for applicability to urban road networks worldwide, regardless of geographical location or infrastructure characteristics. While our experiments focus on two sections of Mexico City, these maps were selected solely for testing the algorithm's performance and convergence behavior. The location and geometric configuration of the maps are not critical to the method itself. The system is generalizable: any user can load a different map, extract its graph, define origin-destination pairs, and run the complete process using the developed code.

The two selected maps exhibit contrasting network structures. One follows a structured, grid-like layout with evenly spaced streets, while the other presents a more organic configuration with winding roads of varying widths. In both cases, intersections serve as decision points for traffic flow and route planning within the algorithm.

This experimental choice was also motivated by the absence of prior studies that address the same optimization problem under the same assumptions.

Although classical benchmark repositories such as the Transportation Networks [26] collection provide valuable instances for traffic assignment studies, they do not include the specific Mexico City areas nor the detailed geometric and operational attributes (e.g., number of lanes, local speed limits, and OSM-based topology) required for our SUMO simulations. Therefore, in this study we generate custom experimental scenarios directly from OpenStreetMap data to ensure consistency between the COS model and the SUMO environment.

To support this, we developed a complete system to automate graph construction, route computation, congestion modeling, and iterative optimization. The codebase includes modules for:

- Parsing origin-destination pairs and mapping them to graph edges.
- Applying a Dijkstra-based routing strategy.
- Computing congestion-adjusted travel times.
- Generating SUMO simulation route files.
- Iteratively refining routes using Best Response Dynamics (BRD).

This modular structure ensures reproducibility and facilitates experimentation on any SUMO-compatible *.net.xml* network.

The first map consists of 382 interconnected nodes covering approximately $0.922$ km$^2$. The second map contains $1536$ nodes over $1.843$ km$^2$. These examples demonstrate the system's capability to handle networks of varying size, complexity, and topology.

Tables I, II, III and IV compares individual and cooperative routing strategies using the following columns

- Indv (T): Total travel time when vehicles follow individually optimal shortest paths.
- Colab (T): Total travel time when routes are computed collaboratively using COS.
- Diff (%): Relative reduction in travel time achieved by cooperation.
- BRD (I): Number of iterations required for BRD to converge to a Nash equilibrium.
- % Congestion: Initial congestion level as a percentage of network capacity, with approximate vehicle count.

In the individual case, each trip independently selects its route by running Dijkstra's algorithm on the graph with edge weights given by the congestion function under the specified initial load level. These individual shortest paths are then kept fixed throughout the experiment and serve as the baseline for comparison with the COS-based collaborative routing.

For all experiments, the reported congestion percentage (% Congestion) represents the *initial* congestion level set at the beginning of each run. This value is independent of the number of trips and serves as a baseline measure of network load relative to its maximum capacity. It is calculated according to the maximum allowable load per edge, as defined in Section 2.6. As the number of trips increases, actual congestion naturally rises, leading to higher saturation levels and potentially longer travel times.

### A. Analysis of Travel Times First Map Scenario

The results in Table I and Fig. 3 show that COS significantly reduces travel times at low to moderate congestion levels (10–50%). For example, at 10% congestion, travel time is reduced by nearly 39%, and at 30% by approximately 31%. These gains demonstrate the algorithm's ability to redistribute traffic efficiently when the network is not saturated.

Beyond 60% congestion, benefits diminish as route alternatives become limited. At 80% or higher congestion, both strategies yield nearly identical results, indicating that the network is operating near capacity and cooperation cannot provide meaningful improvements.

TABLE I
TRAVEL TIMES FOR INDIVIDUAL VS. COOPERATIVE ROUTING UNDER VARYING CONGESTION FIRST MAP SCENARIO, 16 TRIPS

| Indv (T) | Colab (T) | Diff (%) | BRD (I) | % Congestion |
|---|---|---|---|---|
| 42.40 | 25.92 | 38.9 | 33 | 10 ≈ 773 |
| 44.68 | 34.04 | 23.8 | 33 | 20 ≈ 1,547 |
| 66.01 | 45.58 | 30.9 | 33 | 30 ≈ 2,321 |
| 95.99 | 69.52 | 27.6 | 33 | 40 ≈ 3,095 |
| 125.99 | 107.08 | 15.0 | 33 | 50 ≈ 3,869 |
| 141.88 | 137.73 | 2.9 | 50 | 60 ≈ 4,643 |
| 147.98 | 147.79 | 0.2 | 50 | 70 ≈ 5,417 |
| 157.15 | 157.28 | 0.0 | 16 | 80 ≈ 6,191 |
| 159.15 | 159.15 | 0.0 | 16 | 90 ≈ 6,965 |
| 159.17 | 159.17 | 0.0 | 16 | 100 ≈ 7,739 |

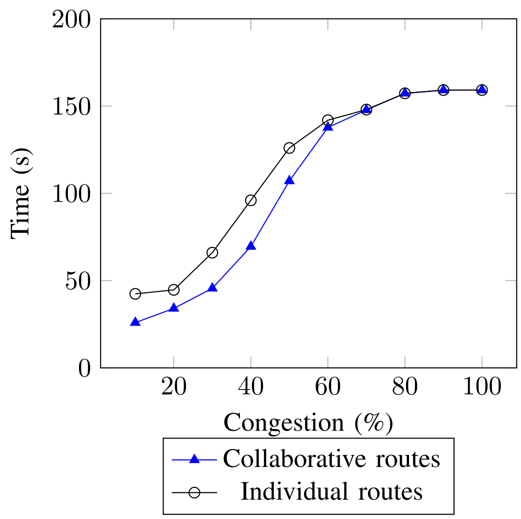


Fig. 3. Individual vs. collaborative routes with COS (16 trips First map).

To evaluate performance under heavier loads, a second experiment used the same map but with 80 trips 16 OD pairs with five vehicles per origin. Results are shown in Table II and Fig. 4.

TABLE II
TRAVEL TIMES FOR INDIVIDUAL VS. COOPERATIVE ROUTING FIRST MAP SCENARIO, 80 TRIPS

| Indv (T) | Colab (T) | Diff (%) | BRD (I) | % Congestion |
|---|---|---|---|---|
| 525.70 | 274.49 | 47.79 | 242 | 10 ≈ 773 |
| 641.07 | 330.67 | 48.42 | 242 | 20 ≈ 1,547 |
| 832.01 | 410.61 | 50.65 | 242 | 30 ≈ 2,321 |
| 973.57 | 538.26 | 44.71 | 242 | 40 ≈ 3,095 |
| 868.68 | 666.06 | 23.33 | 242 | 50 ≈ 3,869 |
| 848.03 | 732.07 | 13.67 | 242 | 60 ≈ 4,643 |
| 773.72 | 767.85 | 0.76 | 242 | 70 ≈ 5,417 |
| 819.03 | 795.08 | 1.85 | 161 | 80 ≈ 6,191 |
| 795.83 | 795.83 | 0.0 | 80 | 90 ≈ 6,965 |
| 795.83 | 795.83 | 0.0 | 80 | 100 ≈ 7,739 |

In this high-load scenario, COS still provides substantial improvements at low and moderate congestion (up to 50%), with reductions exceeding 44%. However, gains decline sharply beyond 60% congestion, and at 70% or higher, results converge, indicating network saturation. An unusual decrease in travel times for individual routes around 50-60% congestion

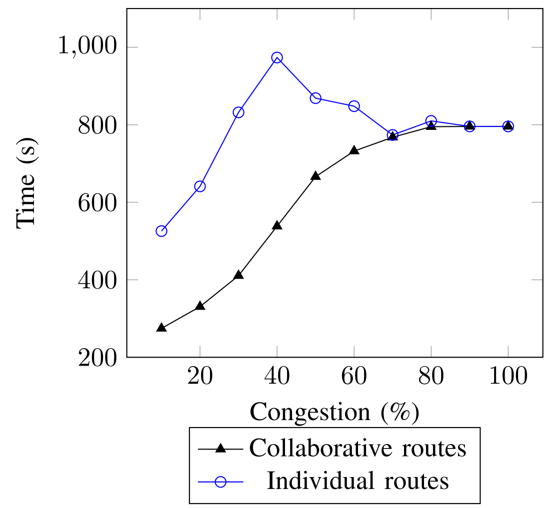


Fig. 4. Individual vs. collaborative routes with COS (80 trips First map).

suggests possible load balancing effects from congestion itself.

### *Analysis of Travel Times Second Map Scenario*

In the first experiment for this scenario, Table III and Fig. 5, we defined 28 unique origin-destination (OD) pairs, assigning one vehicle to each, for a total of 28 trips.

Collaborative routing proved most effective under moderate congestion (10%-60%), with reductions of up to 40% in total travel time compared to individual routing. These gains highlight COS's capability to redistribute traffic efficiently before the network reaches critical saturation.

TABLE III
TRAVEL TIMES FOR INDIVIDUAL VS. COOPERATIVE ROUTING SECOND MAP SCENARIO, 28 TRIPS

| Indv (T) | Colab (T) | Diff (%) | BRD (R) | % Congestion |
|---|---|---|---|---|
| 461.66 | 274.02 | 40.64 | 86 | 10 ≈ 1,394 |
| 577.86 | 354.61 | 38.63 | 57 | 20 ≈ 2,788 |
| 611.66 | 448.41 | 26.69 | 57 | 30 ≈ 4,181 |
| 669.59 | 549.91 | 17.87 | 86 | 40 ≈ 5,575 |
| 723.12 | 645.06 | 10.80 | 57 | 50 ≈ 6,969 |
| 828.15 | 727.44 | 12.16 | 86 | 60 ≈ 8,363 |
| 772.90 | 765.55 | 0.95 | 86 | 70 ≈ 9,757 |
| 770.88 | 770.78 | 0.01 | 57 | 80 ≈ 11,150 |
| 770.94 | 770.94 | 0.00 | 28 | 90 ≈ 12,544 |
| 770.95 | 770.95 | 0.00 | 28 | 100 ≈ 13,938 |

The second experiment retained the same 28 OD pairs but assigned five vehicles per origin, producing 140 trips: Table IV and Fig. 6. This configuration tested COS under higher demand.

For congestion levels between 10% and 40%, COS achieved travel time reductions of 22%-29%. At 60%, gains dropped to around 12%, and at 70%, to just over 1%. Beyond 80% congestion, both strategies converged, with identical travel times at 90% and 100% (3854.74 units), indicating full network saturation.

Best Response Dynamics (BRD) convergence cycles ranged from 422 to 562 for congestion up to 80%, suggesting stable optimization behavior. At 90% and 100%, cycles dropped

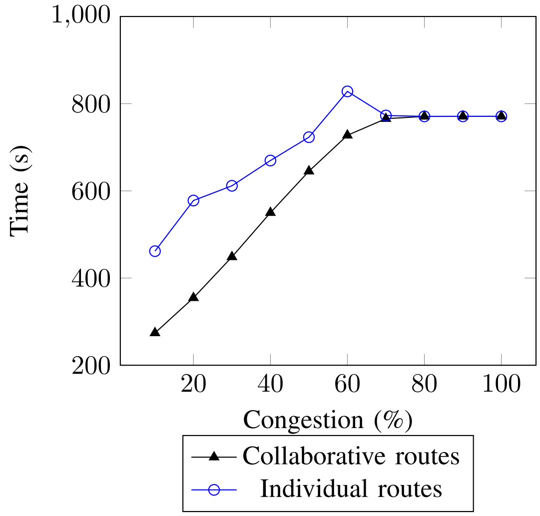


Fig. 5. Individual vs. collaborative routes with COS (28 trips Second map).

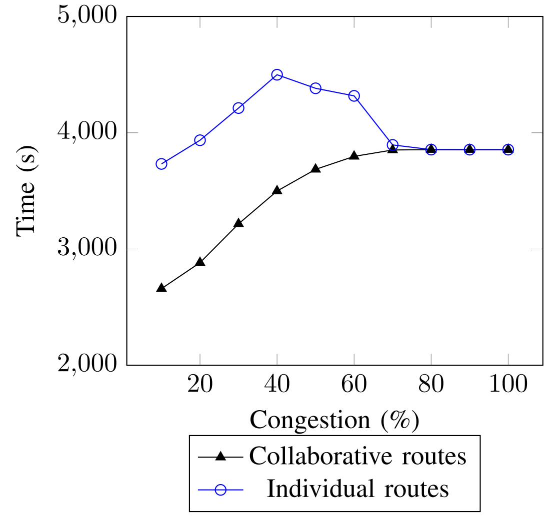


Fig. 6. Individual vs. collaborative routes with COS (140 trips Second map).

sharply to 140, reflecting the absence of further improvement opportunities in fully saturated networks.

These results reaffirm the need for early application of collaborative routing strategies. Once the network approaches saturation, COS's ability to improve performance is minimal.

TABLE IV
TRAVEL TIMES FOR INDIVIDUAL VS. COOPERATIVE ROUTING SECOND MAP SCENARIO, 140 TRIPS

| Indv (T) | Colab (T) | Diff (%) | BRD (I) | % Congestion |
|---|---|---|---|---|
| 3731.77 | 2658.96 | 28.75 | 422 | 10 ≈ 1,394 |
| 3935.48 | 2882.26 | 26.76 | 422 | 20 ≈ 2,788 |
| 4212.37 | 3216.11 | 23.65 | 562 | 30 ≈ 4,181 |
| 4499.07 | 3498.44 | 22.24 | 422 | 40 ≈ 5,575 |
| 4382.35 | 3685.04 | 15.91 | 422 | 50 ≈ 6,969 |
| 4317.23 | 3796.67 | 12.06 | 422 | 60 ≈ 8,363 |
| 3895.31 | 3851.61 | 1.12 | 422 | 70 ≈ 9,757 |
| 3855.11 | 3854.72 | 0.01 | 422 | 80 ≈ 11,150 |
| 3854.74 | 3854.74 | 0.00 | 140 | 90 ≈ 12,544 |
| 3854.74 | 3854.74 | 0.00 | 140 | 100 ≈ 13,938 |

All experiments, COS consistently reduced travel times under low to moderate congestion, with improvements ranging from 20% to nearly 50%. These benefits diminish sharply beyond 70%-80% congestion, where both strategies converge due to network saturation. BRD analysis confirms stable convergence under moderate loads, but rapid, non-beneficial convergence in saturated states. These findings emphasize the importance of proactive deployment of collaborative routing to maximize its impact on urban mobility.

### *B. Individual-level Effects, Route Diversity, and Trade-offs in Collaborative Routing*

While the primary objective of the Collaborative Optimization Strategy (COS) is to minimize total system travel time, it is well known in traffic assignment theory that socially optimized routing does not guarantee improvements for every individual trip. By redistributing traffic across alternative routes to alleviate congestion on critical links, COS may

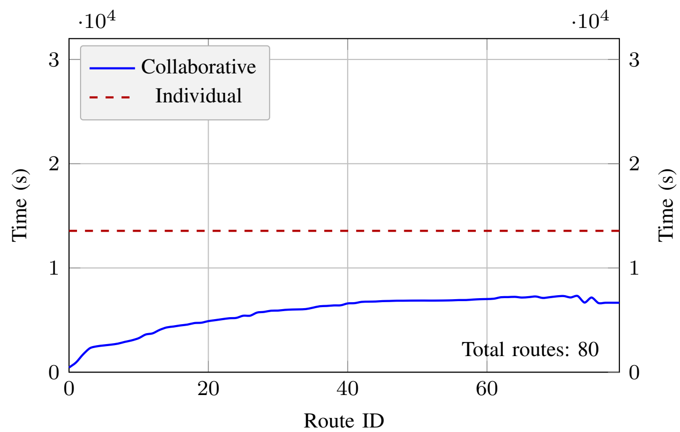


Fig. 7. Collaborative vs. individual routing times under 10% congestion (80 trips, same origin-destination).

intentionally assign a subset of trips to slightly longer or less direct paths in order to achieve a lower global delay. These trips can be interpreted as individual sacrifices required to attain a social optimum, a characteristic inherent to system-level optimization.

This behavior is evident in Figs. Fig. 7-11, which compare collaborative and individual routing times across increasing congestion levels. Under low to moderate congestion (10–30%), collaborative routing exhibits a broader dispersion of travel times across Route IDs relative to the nearly constant individual baseline. This dispersion reflects the exploration of multiple feasible routes and the redistribution of traffic to delay premature convergence to a single dominant path. As congestion increases (50–70%), the dispersion progressively decreases due to network capacity constraints, and under extreme congestion (90%) both approaches converge to similar travel times, indicating that physical limitations dominate routing outcomes.

To provide a per-trip perspective, Figs. 13-15 present routing outcomes in the joint time–distance space. Each point represents an individual routing result, while the red star denotes

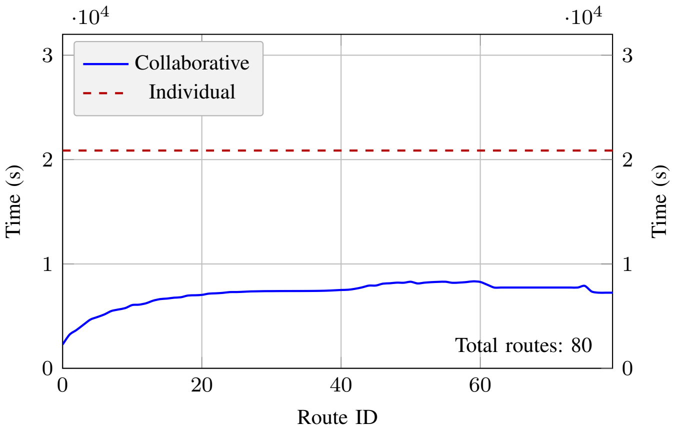


Fig. 8. Collaborative vs. individual routing times under 30% congestion (80 trips, same origin-destination).

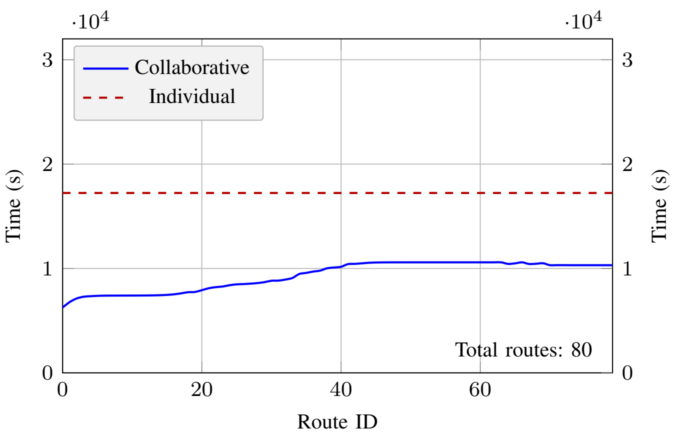


Fig. 9. Collaborative vs. individual routing times under 50% congestion (80 trips, same origin-destination).

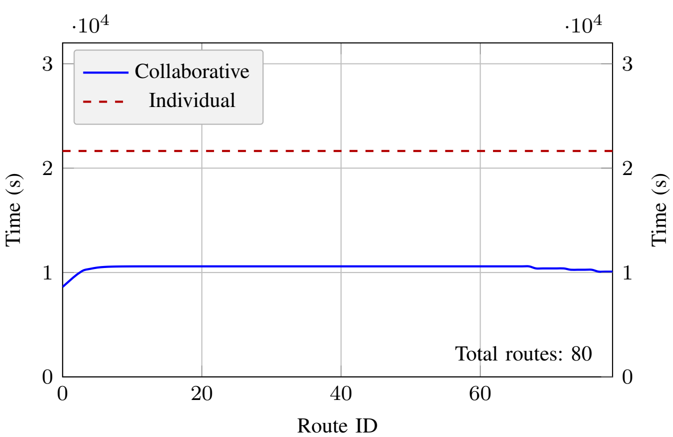


Fig. 10. Collaborative vs. individual routing times under 70% congestion (80 trips, same origin-destination).

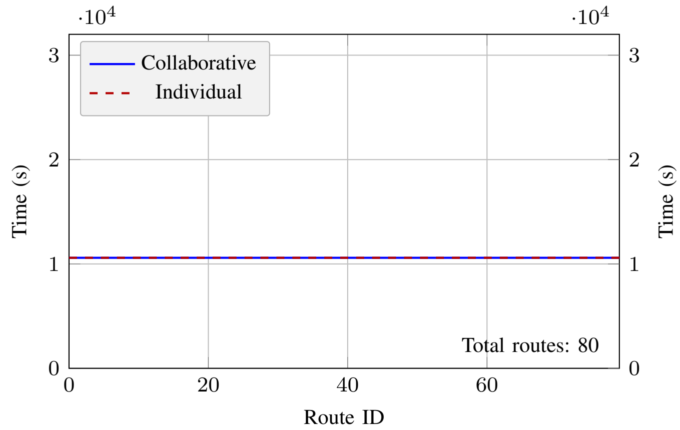


Fig. 11. Collaborative vs. individual routing times under 90% congestion (80 trips, same origin-destination).

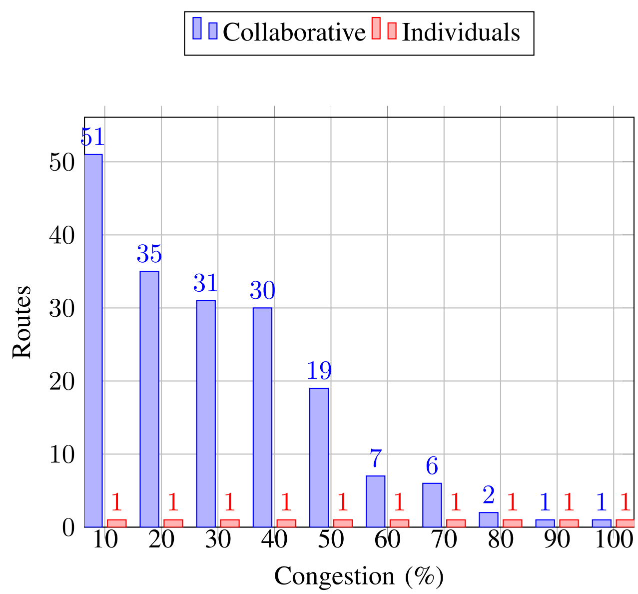


Fig. 12. Comparison of the number of different routes generated by congestion in collaborative vs. individual execution.

the weighted average system behavior. The Euclidean distance between each point and the weighted average captures the deviation of each trip from the collective optimum, jointly accounting for time and distance variations. The gray segments visualize the magnitude of these deviations and serve as a proxy for individual regret. Importantly, even the most distant points remain relatively close to the weighted average, indicating that although some trips are sacrificed, the additional time and distance incurred are bounded and do not result in extreme individual penalties.

Route multiplicity annotations further characterize how gains and losses are distributed. At low congestion, numerous distinct collaborative routes emerge, with multiplicities spread across several solutions, indicating that individual trade-offs are shared among many users rather than concentrated on a small subset. As congestion increases, route diversity decreases and higher multiplicities appear, reflecting convergence toward fewer viable routes and reduced flexibility. Fig. 12 quantifies this effect per congestion level, showing that collaborative routing consistently generates more distinct routes than individual routing. Extending this analysis across all congestion levels and all conducted experiments reveals a total of 132 distinct collaborative routes, compared to only 7 distinct routes under individual routing, highlighting the substantially larger solution space explored by COS.

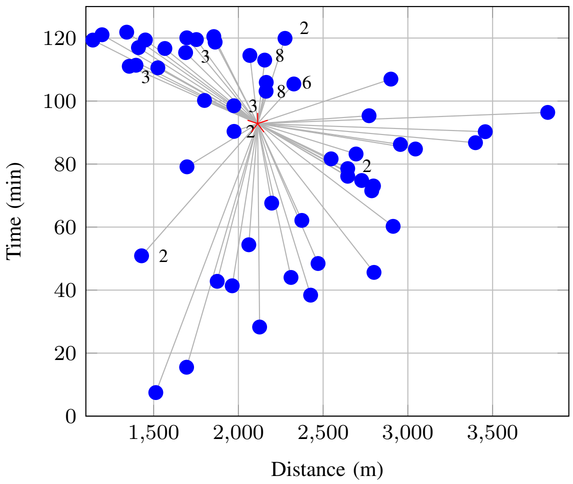


Fig. 13. Euclidean distance of routing outcomes to the weighted average time-distance point under 10% congestion. The red star marks the weighted mean, and blue labels indicate route multiplicity for count $> 1$.

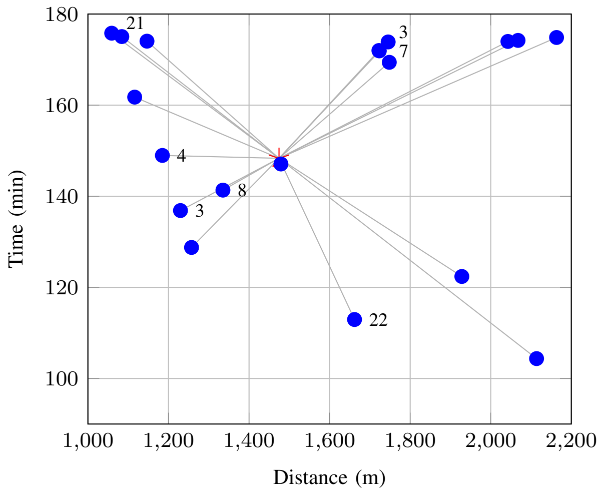


Fig. 14. Euclidean distance of routing outcomes to the weighted average time-distance point under 50% congestion. The red star marks the weighted mean, and blue labels indicate route multiplicity for count $> 1$.

The joint time-distance representation also reveals a correlation between trip length and per-trip outcome. Short-distance trips are more likely to appear farther from the weighted average, indicating that fixed overheads such as detours or additional intersections can dominate perceived cost for these users. In contrast, longer trips tend to cluster closer to the average, suggesting that collaborative routing is more favorable or neutral for them. This observation explains why short-trip users may be less inclined to follow collaborative recommendations, even when overall network performance improves.

Finally, all experiments assume full user compliance, which represents an upper bound on achievable system-level benefits. Nevertheless, the observed route diversity patterns provide insight into partial compliance scenarios. At low to moderate congestion, the presence of multiple alternative routes suggests that partial adherence would reduce but not eliminate system-level gains, as traffic can still be redistributed among feasible paths. At high congestion, where route diversity collapses, partial compliance would have limited impact due to the lack of viable alternatives. While partial compliance is not explicitly modeled, the presented results provide a comprehensive characterization of individual trade-offs, route diversity, and bounded regret under collaborative routing, and establish a foundation for future extensions that incorporate heterogeneous user behavior and adherence dynamics.

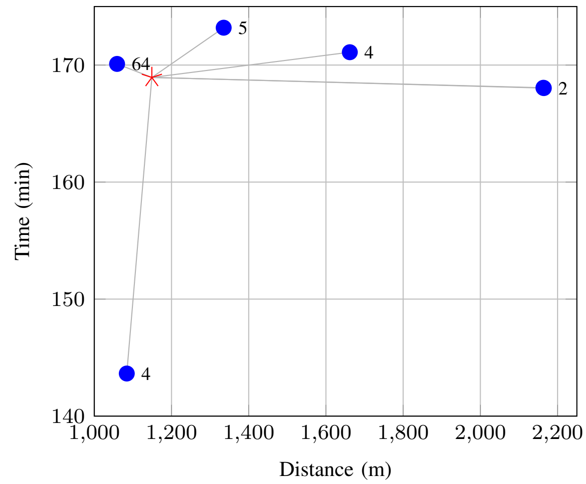


Fig. 15. Euclidean distance of routing outcomes to the weighted average time-distance point under 70% congestion. The red star marks the weighted mean, and blue labels indicate route multiplicity for count $> 1$.

### *C. Analysis of Vehicle Behavior in SUMO Simulation*

SUMO (Simulation of Urban MObility) is a widely adopted microscopic traffic simulator for modeling vehicle movements in complex urban environments [3]. In this study, it served as a key platform for testing the Collective Optimization Scheme (COS) assessing its ability to generate efficient routes while mitigating congestion.

The simulation environment accurately reproduced traffic dynamics across various demand scenarios and congestion levels. SUMO simulations employed the same scenarios generated by COS, reflecting both street congestion levels and the specific trips defined for each experiment. The routes displayed in SUMO represent the outcomes of each trip, obtained through both individual and collaborative COS simulations. Overall, SUMO provides a more realistic environment to observe and analyze the behavior of routes derived from COS.

In our experimental setup, COS and SUMO are tightly coupled. The road network used in SUMO is generated from the same OpenStreetMap data as the graph used by COS, so that the topology, segment lengths, number of lanes, and speed limits coincide. The free-flow travel times and capacity parameters used in the congestion function are derived from these attributes, ensuring that the static edge travel-time estimates approximately match the traversal times observed in SUMO under low congestion. COS is first used to compute a fixed route for each trip; these routes are then imported into

SUMO and assigned to the corresponding vehicles. SUMO does not modify the routes, but simulates the microscopic dynamics along the COS-computed paths. This setup allows us to qualitatively validate that collaborative routing results in more dispersed use of the network and mitigates localized congestion under a realistic dynamic traffic model.

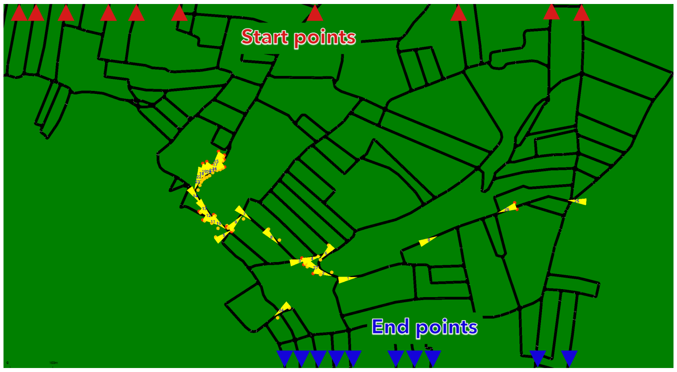


Fig. 16. Vehicle behavior under individual routing strategy in SUMO.

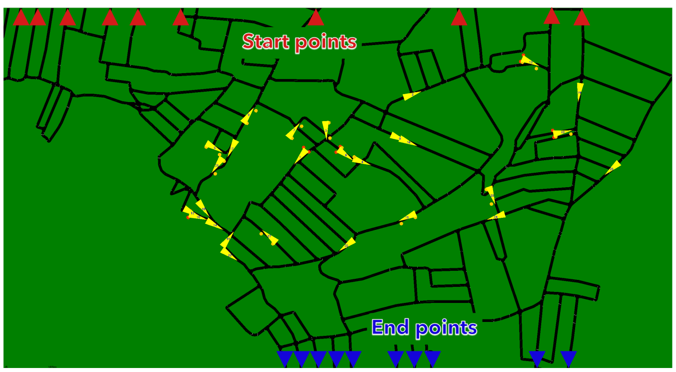


Fig. 17. Vehicle behavior under collaborative routing strategy in SUMO.

In Fig. 16, individually optimized routes concentrate traffic along a few primary corridors. While efficient in light traffic, this behavior quickly produces localized congestion under higher demand. By contrast, Fig. 17 shows that COS-based collaborative routing distributes vehicles more evenly across available paths, reducing load on critical segments and improving overall network balance. This redistribution results from the algorithm's adaptive nature, which updates routes in response to real-time congestion conditions.

## V. Related Work

This section situates our methodology within the broader traffic management literature, highlighting its novelty and practical contributions. While many models aim to optimize vehicular mobility, few adopt a collaborative approach that balances individual preferences with network-wide efficiency. To our knowledge, no prior studies combine the objectives, framework, and experimental design presented here. Our approach integrates Best Response Dynamics (BRD) with Dijkstra's algorithm to promote cooperation, reducing overall congestion rather than optimizing routes in isolation.

Popular navigation platforms such as Waze and Google Maps use real-time and historical data to improve routing [10], [13]. Waze relies on crowdsourced reports, which can be inconsistent, while Google Maps leverages GPS data and algorithmic enhancements. Both systems prioritize individual optimization, potentially causing collective inefficiencies. In contrast, COS distributes vehicular flow strategically across the network, mitigating bottlenecks without requiring user-generated

Cabannes [5] highlights that individual route optimization can exacerbate congestion, advocating for collaborative strategies, supporting COS's motivation. Similarly, bio-inspired and predictive models [16], [29] and discrete dynamic models differentiating globally and locally informed users [12] emphasize that intelligent, cooperative routing can reduce congestion more effectively than infrastructure expansion alone.

Li [11] extend Dijkstra's algorithm to compute comfort-based routes by weighting the road network with variables such as roughness indices, anomaly costs, and intersection penalties. Although their approach generates routes individually, this integration of contextual factors represents a valuable contribution. In contrast, our Collective Optimization Scheme (COS) employs a game-theoretic framework to promote cooperative routing and mitigate congestion at the network level.

Classical work in traffic assignment has highlighted the contrast between user-equilibrium and system-optimal solutions. For example, LeBlanc and Abdulaal compare user-optimum versus system-optimum traffic assignment in a transportation network design context, showing that system-optimal formulations can yield solutions of comparable quality with more favorable computational properties [14]. More recently, Morandi provides a comprehensive review of approaches that bridge user equilibrium and system optimum in static traffic assignment through hybrid and coordinated mechanisms [17]. From a game-theoretic perspective, Satsukawa et al. formulate dynamic system optimal traffic assignment with atomic users as a strategic game and analyze the convergence and stability of various evolutionary dynamics [24]. In this context, the proposed COS framework can be viewed as a coordinated routing scheme for atomic users over a congestion-dependent network, leveraging best-response dynamics to approach system-level objectives while explicitly accounting for individual route choices.

Unlike most prior work focused on individual or heuristic-based routing, the proposed Collective Optimization Scheme (COS) introduces a structured, game-theoretic framework for cooperative route optimization in congestion-dependent networks. By integrating Best Response Dynamics with Dijkstra's algorithm, COS explicitly balances individual route choices with network-wide efficiency, mitigating congestion through coordinated traffic redistribution rather than isolated optimization.

Compared to popular navigation platforms that prioritize user-centric routing, COS targets system-level performance while preserving bounded individual trade-offs. The approach aligns with classical distinctions between user-equilibrium and system-optimal traffic assignment and leverages best-response dynamics to ensure convergence to stable solutions. Experimental validation using SUMO confirms that COS increases route diversity under low to moderate congestion, distributes individual sacrifices across users, and converges to stable outcomes under high congestion, demonstrating its scalability, adaptability, and practical relevance for urban traffic management

## VI. Conclusions

COS was developed to minimize total travel time for a set of trips by integrating Best Response Dynamics with Dijkstra's algorithm. By generating cooperative routes, COS leverages available road network capacity more effectively and fosters collaboration among participants. Simulations based on sections of Mexico City's road network demonstrate that the scheme is both versatile and globally applicable, offering a promising strategy for addressing urban traffic congestion.

Experimental results confirm that COS can significantly reduce total travel time by distributing vehicular flow more evenly across the network, particularly under low to moderate congestion levels. As congestion intensifies and the network approaches saturation, the benefits of collaborative routing diminish, eventually aligning with those of purely individual routing due to physical infrastructure constraints.

These findings highlight the importance of implementing collaborative routing strategies proactively, before congestion reaches critical thresholds. COS's capacity to balance traffic across alternative paths contributes directly to improved network efficiency and urban mobility. Moreover, the analysis of Best Response Dynamics indicates stable convergence under moderate congestion, underscoring the robustness of the proposed decision-making framework.

The congestion function proved essential for accurately modeling street-level traffic flow; however, further refinements are needed to capture additional real-world complexities such as intersection dynamics and street-priority rules. Incorporating these factors represents a key avenue for improving the realism and adaptability of future routing strategies.

From a computational standpoint, COS remains tractable. The use of Dijkstra's algorithm ensures polynomial-time performance for individual path computations, while the iterative Best Response Dynamics converge efficiently in practice for moderately sized networks. Although complexity scales with the number of agents and network size, simulations suggest that COS is computationally feasible for offline strategic planning. With additional optimization and parallelization, the framework could be adapted for near real-time applications. Enhancing scalability and integrating richer traffic control mechanisms constitute important directions for future research.

## Acknowledgements

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